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THE

PHYSICAL REVIEW.

THE THERMIONIC CURRENT BETWEEN PARALLEL PLANE ELECTRODES; VELOCITIES OF EMISSION DISTRIBUTED ACCORDING TO MAXWELL'S LAW.

By Thornton C. Fry.

Synopsis.

Thermonic Current between Parallel Plane Electrodes.—The electrical equations applying to this problem are developed without neglecting the distribution of initial velocities, which in the first place is allowed to be entirely general. *Maxwell's distribution of velocities* is then considered in detail and a complete solution obtained.

Curves from which to compute the space current when Maxwell's distribution applies are presented, together with an illustrative example of their use. Curves are also included showing the *deviation of the current-voltage relation from the 3/2-power law;* the variation of the *minimum potential between the electrodes* with plate voltage, and also the variation with plate voltage of *the distance between the cathode and the point at which this minimum potential occurs.*

1. Introduction.

A SIMPLE method of computing the thermionic current between parallel plane electrodes under circumstances such that the 3/2-power law published by Childs[1] and later by Langmuir[2] does not apply seems highly desirable. An attempt made several years ago to solve this problem led to a method of solution which was at once direct and general. It is the aim of this paper to present this method of solution as concisely as possible; and to apply it to a particular problem with a view to deriving quantitative results of a typical sort.

The solution requires the evaluation of a difficult integral which appears in equation (4). This evaluation was carried out, with the result shown in Figs. 2 and 4, by the use of the Integraph of Abdank-Abakanowicz. This integraph, which is not so well known in this country as its relatively low cost and very great accuracy seem to justify, is so constructed that when a pointer is passed along a curve $y = f(x)$ a ruling pen attached to the machine automatically draws the curve representing

[1] Phys. Rev., 32, p. 492, 1911.

[2] Phys. Rev., 2, p. 350, 1913.

the indefinite integral, $\int f(x)dx$. Its accuracy when properly operated is such that successive integrations of the same function will seldom vary from one another by more than the width of the line drawn by the ruling pen.

2. MATHEMATICAL FORMULATION.

It is obvious that in any case of thermionic emission in which voltage saturation has not been reached there must be a region of negative potential gradient adjacent to the cathode. If this were not true every emitted electron would be drawn to the anode, and consequently no increase of current would result from an increase in the anode voltage. It therefore follows that the voltage curve must have the characteristic form shown in Fig. 1. A convenient notation denotes the potential at an arbitrary distance x from the cathode by V, the minimum potential by V' and the distance of this minimum potential from the cathode by x'. The region between O and x' may be called the region α, the remainder of the distance to the anode being denoted by β. If attention is fixed upon a particular electron its emission velocity may be denoted by v_0, its velocity at x' by v' and its velocity at an arbitrary point x by v. The number of electrons emitted per second per unit area with this particular velocity v_0 may be called $n(v_0)$ and the total emission N. Then

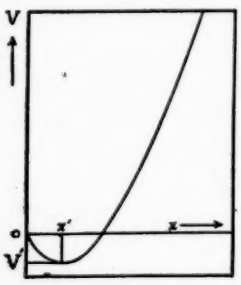

Fig. 1.
Characteristic Form of Voltage Curve.

$$N = \int_0^\infty n(v_0)dv_0.$$

In a current of electrons all of which travel with the speed v, the space charge is given by $e[n(v_0)/v]$, $n(v_0)$ being the number of them emitted per unit time. Hence where velocities differ the space charge is given by the equation

$$\rho = e\int \frac{n(v_0)}{v}\,dv_0,$$

the integration being performed over all the velocities of emission which are of such a magnitude that the electrons pass the point in question.

If this point lies in the region β all electrons pass it which had emission velocities greater than $\sqrt{2V'e/m}$, that is, high enough to pass the region of adverse gradient. Hence, denoting this critical emission velocity by v_0',

$$\rho = e\int_{v_0'}^\infty \frac{n(v_0)}{v}\,dv_0. \qquad (1)$$

The space charge at a point x in α is not quite so simply expressed, since it is made up of those electrons which pass on to the anode and also of those which, although possessing enough energy when emitted to pass x, are brought to a stop before reaching x' and turn back. There are two equally dense streams of these latter electrons passing in opposite directions, each of which contributes to the space charge. If V is the potential of the point whose distance from the cathode is x, those electrons which are emitted with lower velocities than $\sqrt{2Ve/m}$ do not reach x, and those which are emitted with velocities higher than $v_0' = \sqrt{2V'e/m}$ do not return after passing. Hence,

$$\rho_\alpha = 2e\int_{\sqrt{\frac{2Ve}{m}}}^{v_0'} \frac{n(v_0)}{v}\,dv_0 + e\int_{v_0'}^{\infty} \frac{n(v_0)}{v}\,dv_0. \tag{2}$$

These formulæ assume that all the electrons are shot out normally to the cathode. They are equally true, however, whatever the direction of emission may be, provided v_0 is understood to signify the *normal component* of the emission velocity and $n(v_0)$ the number of electrons shot off per second with this *normal component*. When later in the paper the Maxwell distribution of velocities is introduced the normal component only, and not the complete velocity, will be dealt with.

The current to the cathode is given by

$$\frac{i}{e} = \int_{v_0'}^{\infty} n(v_0)dv_0. \tag{3}$$

The only other equations necessary to determine the solution of the problem are the equation of energy,

$$v^2 = v_0^2 - \frac{2Ve}{m}, \tag{4}$$

and Poisson's equation,

$$\frac{\partial^2 V}{\partial x^2} = -4\pi\frac{\rho}{k}, \tag{5}$$

k being the dielectric constant.

Equations (1) and (5) of section 2 result in the differential equation:

$$\frac{d^2V}{dx^2} = -\frac{4\pi e}{k}\int_{v_0'}^{\infty} \frac{n(v_0)}{v}\,dv_0,$$

which formulates the potential distribution in the region β. A first integral of this equation is found in the customary manner by multiplying it, on both sides, by $2(dV/dx)$, and integrating the right-hand side under the sign of integration, the result being,

$$\left(\frac{dV}{dx}\right)^2 = \frac{8\pi m}{k}\int_{v_0'}^{\infty} n(v_0)(v - v')dv_0. \tag{6}$$

This result applies only to the region β. By using (2), (4) and (5) in exactly the same way a similar result may be obtained for the region α, provided the limits of integration, which are variable in this case, are properly dealt with when the order of integration is changed. This gives:

$$\left(\frac{dV}{dx}\right)^2 = \frac{8\pi m}{k}\left[\int_{v_0'}^{\infty} n(v_0)(v - v')dv_0 + 2\int_{\sqrt{\frac{2Ve}{m}}}^{v_0'} vn(v_0)dv_0\right]. \qquad (7)$$

So far, it has not been necessary to know the form of the function $n(v_0)$, but in order to proceed further it is desirable to introduce it at this time. The most logical expression is Maxwell's distribution law for the number of gas molecules having the normal velocity v_0 which pass a given plane in a second of time. This expression is

$$n(v_0) = \frac{\pi N v_0}{2\bar{v}_0^2}\,\epsilon^{-\left(\frac{\sqrt{\pi}v_0}{2\bar{v}_0}\right)^2}, \qquad (8)$$

in which $\bar{v}_0$ represents the average velocity of emission,

$$\bar{v}_0 = \frac{1}{N}\int_0^{\infty} n(v_0)v_0 dv_0.$$

It is somewhat simpler to use v instead of v_0 as the variable of integration in (6) and (7). When this is done and $n(v_0)$ is given the value (8) the equation

$$\left[\frac{dV}{dx}\right]^2 = \frac{4\pi Nm}{k\bar{v}_0^2}\left[\int_0^{\infty} v^2\epsilon^{-\frac{\pi}{4\bar{v}_0^2}\left(v^2+\frac{2Ve}{m}\right)}dv - \int_{\sqrt{\frac{2e}{m}(V'-V)}}^{\infty} vv'\epsilon^{-\frac{\pi}{4\bar{v}_0^2}\left(v^2+\frac{2Ve}{m}\right)}dv \pm \int_0^{\sqrt{\frac{2e}{m}(V'-V)}} v^2\epsilon^{-\frac{\pi}{4\bar{v}_0^2}\left(v^2+\frac{2Ve}{m}\right)}dv\right] \qquad (9)$$

is obtained, in which the upper sign is to be used in the region α and the lower sign in the region β. The integrals involved in this expression are of well-known types and may easily be evaluated either in terms of algebraic functions or in terms of the normal error function. Denoting this latter function by

$$\text{erf}\,(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx,$$

(9) may be expressed as

$$\left(\frac{d\eta}{d\xi}\right)^2 = \epsilon^{\eta} - 1 \pm \left(\epsilon^{\eta}\,\text{erf}\sqrt{\eta} - \frac{2}{\sqrt{\pi}}\sqrt{\eta}\right), \qquad (10)$$

where the quantities ξ and η are related to x and V by the formulæ

$$\xi = \epsilon^{-\frac{\pi eV'}{4\bar{v}_0^2 m}}\sqrt{\frac{2\pi^2 e^2 N}{km\bar{v}_0^3}}\,(x - x'),$$

$$\eta = \frac{\pi e}{2\bar{v}_0^2 m}\,(V - V').$$

It is to the introduction of these quantities ξ and η that the generality of the solution which we obtain must be attributed. They are both dimensionless, but, as the defining equations show, they are proportional to distance and potential difference respectively, both being measured from the point of minimum potential. Hence for any one state of the system the curve which represents η as a function of ξ will also represent V as a function of x; the only difference being that for the latter purpose the units of measure along the axes, and the location of the origin of reference must be changed. For this reason it is probably most convenient physically to think of ξ and η as length and potential respectively, remembering, however, that with each change in such quantities as cathode temperature and anode voltage the unit of measure must be readjusted.

From a purely mathematical point of view the reason it is desirable to use them rather than x and $\bar{v}$ is that they reduce equation (9) to the form (10) in which, aside from themselves, only absolute constants occur. Equation (9) defines one relation between V and x for every set of values of the constants $\bar{v}'$, $\bar{v}_0$, etc., and therefore to obtain a complete solution for it by graphical means would require the construction of a large number of curves. On the other hand equation (10) has only one solution which may be found once for all. The labor of obtaining a complete answer to our problem is, therefore, materially reduced by the use of the new symbols.

3. Solution of Equation (10).

The equation (10) applies to the region between the cathode and the point of minimum potential when the upper sign is used and to the region beyond the point of minimum potential when the lower sign is used. In either case, however, the boundary condition to be applied is that $V = V'$, and hence $\eta = 0$ when $x = x'$, and hence $\xi = 0$. The right-hand side of this equation has been computed for a sufficient range of values of η. These values are shown in the accompanying table, where it is denoted by $\phi(\eta)$.

The equation (10) may now be written in the form

$$\xi = \int_0^\eta \frac{d\eta}{\sqrt{\phi(\eta)}}, \tag{11}$$

which represents the general solution of the problem under discussion provided the integral can be evaluated. This could easily be accomplished by the use of the integraph if it were not for the fact that the integrand becomes infinite at $\eta = 0$. However, no mechanical device is ever capable of handling infinities, and it is necessary to perform the

TABLE I.

Values of the Functions $\phi(\eta)$.

η	$\phi(\eta)$.		$[\phi(\eta)]^{-1/2}$.	
	Upper Sign.	Lower Sign.	Upper Sign.	Lower Sign.
.1	.129	.0808	+2.79	−3.52
.2	.293	.1488	1.85	2.59
.3	.489	.2107	1.43	2.19
.4	.716	.2681	1.06	1.93
.5	.977	.3214	1.01	1.76
.6	1.271	.3726	.887	1.64
.7	1.607	.4213	.789	1.54
.8	1.983	.4688	.710	1.46
.9	2.408	.5118	.644	1.40
1	2.878	.5575	.590	1.34
1.2	4.002	.6377	.500	1.25
1.4	5.384	.716	.431	1.18
1.6	7.116	.790	.375	1.12
1.8	9.24	.862	.329	1.08
2	11.85	.931	.291	1.04
2.5	21.27	1.09	.217	.962
3	37.13	1.24	.164	.901
3.5	62.85	1.38	.126	.855
4	105.68	1.52	.097	.813
4.5	176.41	1.63	.075	.781
5	293.07	1.75	.058	.758
9		2.56		0.625
16		3.65		0.525
36		5.86		0.413
64		8.09		0.352
100		10.34		0.311
225		15.9		0.251
400		21.6		0.215
900		32.8		0.174
1600		45.2		0.149
2500		56.4		0.133

integration for small values of η in some other way. This is done by noting that when η is small

$$\phi(\eta) = \eta \pm \frac{2}{\sqrt{\pi}}\eta^{3/2} + \frac{1}{2}\eta^2.$$

Substituting this value in (11) and performing the integration the approximate relation

$$\xi \doteq 2\sqrt{\eta} \mp \frac{1}{\sqrt{\pi}}\eta + \left(\frac{3}{\pi} - \frac{1}{2}\right)\eta^{3/2} \qquad (12)$$

is obtained.

This curve is shown plotted as curve (1) of Fig. 2. Curve (2) is $[\phi(\eta)]^{-1/2}$ for the region between the cathode and the point of minimum potential while curve (3) represents the same function beyond this point. Curves (4) and (5) are the integrals of (2) and (3) as drawn by the integraph, so constructed as to meet the approximation curve (1) near the origin. From the fact that (4) and (5) join on to (1) smoothly, it is evident that the approximation given by (12) is sufficiently accurate for all physical purposes.

4. Discussion of Results.

There are three distinct types of potential distribution curves, for the case of pure electron discharge between plates. The first of these is met when the space current is saturated; that is, when every electron which frees itself from the cathode is pulled across to the anode. It is at once obvious that the potential gradient in this case cannot be negative; hence this curve must have a positive slope throughout its entire range. It is typified by curve (1) of Fig. 3. As the plate voltage is increased,

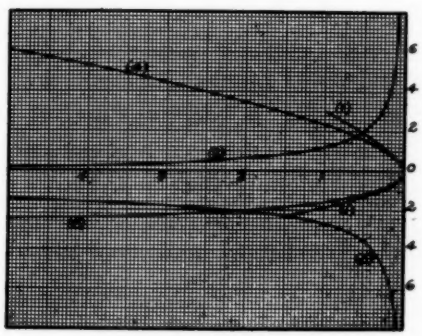

Fig. 2.
Voltage Distribution Curve, Approximation Curve, and Derivatives.

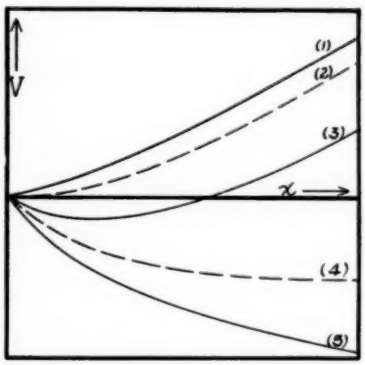

Fig. 3.
Three Types of Voltage Distribution.

it straightens out more and more, and the angle at which it meets the cathode becomes more and more acute. Similarly, as the plate voltage decreases toward the critical voltage which will just saturate the current the curve rounds off, and the angle at which the cathode is met becomes more and more nearly a right angle. When the plate voltage is made just sufficient to saturate the current a limiting distribution of potential is met that separates the first type of curve from the second. For this distribution the gradient at the surface of the cathode is zero, while at all other points of space it is positive. Such a curve is shown by the dotted line (2) of Fig. 3.

For all distributions of potential belonging to the first class, the space current is constant and determined solely by the emissivity of the cathode. Whatever the design of the tube or the voltage applied, so long as it exceeds the saturation voltage, this current remains constant. When, however, the voltage is reduced below the limiting value of this class the gradient at the cathode, and for some distance x' beyond, becomes negative. At x' it changes sign and thereafter remains positive, as shown in Fig. 3 (3).

At x' there is a minimum on the potential distribution curve. The value of V' at this minimum point, and the distance x' between it and the cathode vary with the anode potential. When this is so high that nearly all of the emitted electrons are drawn across both V' and x' are very small; but as it is lowered more and more V' becomes greater and greater, and moves further and further from the cathode. During these changes of plate voltage the current is determined solely by the number of electrons which are capable of passing the restraining voltage V', the relation between them being that given in equation (3).

When the anode voltage is made zero the potential throughout the space between the plates is everywhere negative, and this remains true as the anode voltage is still further reduced. A second limiting condition, which separates the second and third classes of potential distributions is reached when the anode potential is so low that x' becomes equal to the distance between the plates. When this condition is reached the potential gradient at the anode is zero; while throughout the remainder of the space between the electrodes it is negative. The voltage on the anode is then V', and the space current is determined by the number of electrons emitted with sufficient energy to overcome this potential drop. This limiting condition is shown in Fig. 3 by a dotted line (4).

When the potential on the anode is still further reduced, the gradient becomes everywhere negative, and curves of type (5) result. When potential distributions of this type occur the space current is determined solely by the number of electrons which are able to move against the adverse potential $-V$.

The second condition is the one most frequently met in practice, and is the only one to which consideration is given in this paper. The curve obtained in Fig. 2 when drawn for a sufficient range of values of ξ and η as in Fig. 4 applies to all distributions of this type and makes possible computations regarding them. In order to make use of it most conveniently, however, it is desirable to express ξ and η in terms of quantities which are not so difficult to determine as V' and $\bar{v}_0$. The current i, the saturation current i_s and the potential change $\overline{V}_0$ which would give

to an electron an energy equal to the average energy of those shot out from the cathode are satisfactory in this respect, and it is not difficult

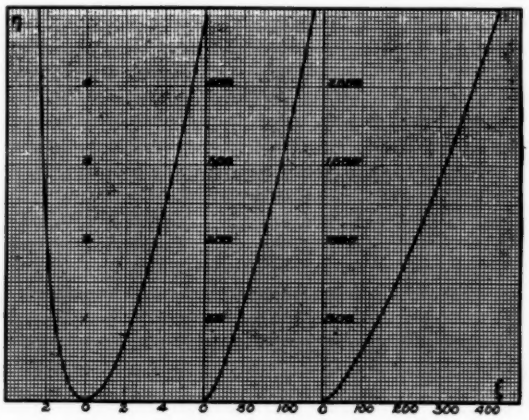

Fig. 4.

The ξ, η Curves.

in the light of the preceding equations to see that they are related and the quantities $\bar{v}_0$ and V' by means of the equations

$$i_s = Ne,$$

$$\overline{V}_0 = -\frac{2m}{e}\bar{v}_0^2,$$

$$i = i_s \epsilon^{V'/\overline{V}_0}.$$

The last of these equations furnishes the relation between V' and i, which can be written in the form

$$V' = \overline{V}_0 \log_\epsilon \frac{i}{i_s}. \tag{13}$$

Upon introducing these new symbols in the equations defining ξ and η and inserting the numerical values of such of the quantities as are universal constants, they become

$$\begin{aligned} \xi &= 820 \frac{i^{1/2}}{\overline{V}_0^{3/4}}(x - x'), \\ \eta &= \frac{V - V'}{\overline{V}_0}, \end{aligned} \tag{14}$$

where current and potential are expressed in amperes and volts.

In order to illustrate the procedure to be followed in making use of Fig. 4 a particular example may be considered. For this purpose take a pair of electrodes $\frac{1}{2}$ cm. apart, one of which is emitting electrons in such

a fashion that[1] i_s is 0.16 amperes and $\overline{V}_0$ is 0.3 volts. In this case if ξ' and η' are the values of ξ and η at the cathode equations (14) become

$$\eta' = \log \frac{i_s}{i},$$
$$V = 0.3(\eta - \eta'), \tag{15}$$
$$\xi = \xi' + 1010\sqrt{i}.$$

In Table II. a set of values of i has been chosen ranging from 0.1 per cent. to 100 per cent. saturation. From these values η' has been found

TABLE II.

The Application of Fig. 4 to a Specific Problem.

i/i_s.	i amps.	η'.	ξ'.	$1010\sqrt{i}$.	ξ.	η.	V Volts.	x' cm.	V' Volts.
0.001	0.00016	6.91	−2.4	12.8	10.4	13	2	0.094	−2.1
0.002	0.00032	6.21	−2.4	18.0	15.6	23	5	0.067	−1.9
0.005	0.0008	5.30	−2.4	28.4	26.0	47	13	0.042	−1.6
0.01	0.0016	4.61	−2.40	40.4	38.0	80	23	0.030	−1.4
0.02	0.0032	3.91	−2.35	57.1	54.8	133	39	0.021	−1.2
0.05	0.008	3.00	−2.24	90.4	88.2	263	78	0.012	−0.9
0.1	0.016	2.30	−2.09	128	126	435	130	0.008	−0.7
0.2	0.032	1.61	−1.88	180	178	695	208	0.005	−0.5
0.4	0.064	0.92	−1.56	255	253	1130	338	0.003	−0.3
0.6	0.096	0.51	−1.22	313	312	1515	453	0.002	−0.2
0.8	0.128	0.22	−0.86	361	360	1840	550	0.001	−0.1
1.0	0.16	0.00	−0.00	404	404	2150	645	0.000	−0.0

by the first of equations (15) and then ξ' taken from the curve of Fig. 4. Knowing the values of ξ', it is easy to compute the value of ξ corresponding to each value of i. Using these values of ξ, η is picked off from Fig. 4 and entered in the table. Finally V is obtained by substituting in the second of equations (15) the values already found for η and η'.

The relation between V and i is represented graphically in Fig. 5, where it has been plotted to logarithmic scales in order to facilitate comparison with the 3/2-power law, which gives the accompanying straight line.

In the process of obtaining this curve it has been necessary to find the values of ξ' and η', from which x' and V' can easily be obtained by the use of the relations $V' = -0.3\eta'$ and

$$x' = -\frac{\xi'}{2020\sqrt{i}}.$$

Accordingly these quantities, which have a certain amount of interest in themselves, have been entered in the last two columns of Table II. and

[1] This corresponds roughly to a tungsten cathode at 2400° K.

represented by Figs. 6 and 7. Of course the absolute magnitudes of x' and V' corresponding to any given voltage are very largely dependent upon such things as the electrode separation and the saturation current,

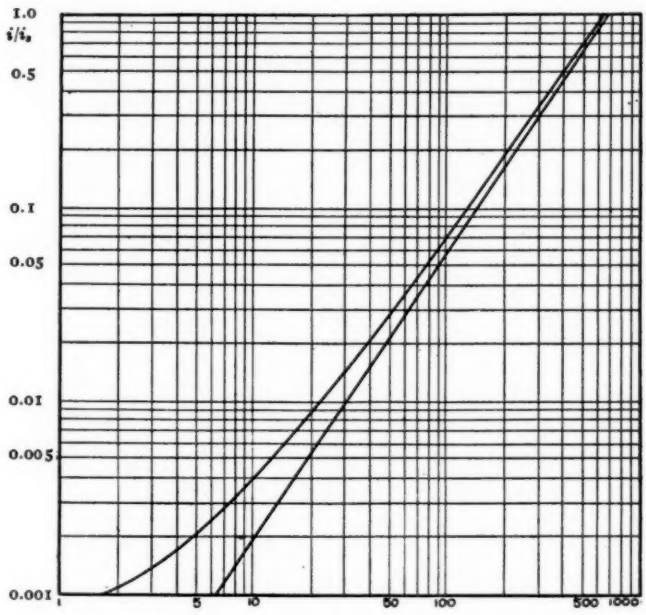

Fig. 5.

The Relation of Space Current to Plate Voltage: Comparison of Maxwell Distribution and 3/2-Power Law.

so that Figs. 6 and 7 can be regarded as typical only in the sense that curves of the same general shape are obtained for other values of these quantities.

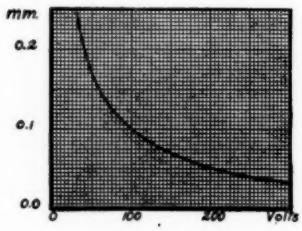

Fig. 6.

The Relation between Plate Voltage and Distance of Minimum Potential from Cathode.

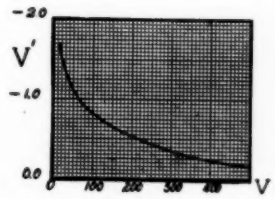

Fig. 7.

Relation between Minimum Potential and Plate Voltage. The Scales are in Volts.

5. Conclusion.

Perhaps the most interesting comment which may be made on these results concerns the magnitude of the variations from the 3/2-power law

introduced by the finite velocities of emission. These variations are frequently quite large, and, as is evident from Fig. 5, may be close to 50 per cent. for voltages as high as 40 or 50 volts. At still lower voltages the 3/2-power law need not even give a rough approximation to the values to be expected from a Maxwell distribution; so that at these voltages a more accurate means of computation is needed. This means is furnished by the curves of Figs. 2 and 4, the use of which, it is believed, will be found sufficiently simple and accurate for most laboratory computations.

RESEARCH LABORATORIES OF THE AMERICAN TELEPHONE &
TELEGRAPH COMPANY AND THE WESTERN ELECTRIC COMPANY, INC.,
February, 1920.

FLAME EXCITATION OF LUMINESCENCE.

BY E. L. NICHOLS AND D. T. WILBER

SYNOPSIS.

Luminescence Excited by Contact with the Hydrogen Flame.—It has been discovered that many substances may be excited to fluorescence by contact with a hydrogen flame. The conditions for this excitation are described and it is shown that this luminescence differs from photo-luminescence and also from mere temperature radiation.

Many pure oxides and sulphides not luminescent under the action of light (notably the oxides of calcium, magnesium, silicon, aluminum, zirconium and zinc and the sulphides of zinc and calcium) and certain of the phosphorescent sulphides are shown to respond to flame excitation.

The range of temperatures through which flame excitation occurs is perfectly definite for each substance and the upper limit is sometimes several hundred degrees above that at which photo-excitation becomes extinct.

The spectra, like those of the phosphorescent sulphides, are broad banded and when subjected to spectro-photometric measurements they are found to consist of numerous equidistant overlapping components. These components, in the case of substances capable of photo-luminescence are identical with the components of the fluorescence spectrum excited by light but are of different relative intensities.

THE existence of a definite well-defined upper limit of temperature above which bodies are neither phosphorescent nor fluorescent has long been accepted in the science of luminescence. Lenard and Klatt[1] in an early paper estimated roughly the temperatures of extinction of the afterglow and of fluorescence for the phosphorescent sulphides. The "upper momentary-range" in their scheme of temperature relations lies between the extinction temperature of phosphorescence and the still higher temperature at which fluorescence ceases.

The precise boundaries of this region were left undetermined, but their tables show fluorescence to be in general very dim or entirely gone at 400°. In the case of Ca-Bi; Sr-Cu; Sr-Zn and Sr-Bi they were still able to detect traces of it when the substance was at a dull red heat. Phosphorescence usually disappears at a much lower temperature (200° to 300°).

There are however instances of visible radiation at somewhat higher temperatures which are suggestive of luminescence and the study of one of these, in particular, was the starting point of the present investigation.

When the oxygen of an oxy-hydrogen blast lamp used in the production

[1] Lenard and Klatt, Annalen der Physik, XV., p. 425 (1904).

of the lime light is turned off and the hydrogen flame is allowed to play over the surface of the slowly cooling lime cylinder, a greenish glow may sometimes be seen in the regions reached by the flame. This furtive glow which occurs at temperatures corresponding to a very low red heat, or below, must have been familiar to many of the early users of the projection lantern. It has however never been studied or described, so far as we are aware.

Attempts to produce this effect in somewhat modified form led to a number of striking results. In our first experiments air-slaked lime from old lime light cylinders which had broken down under exposure to the atmosphere was inserted in a hydrogen flame.

For convenience the powder was placed on a fine wire gauze and rubbed into the mesh. It was immediately noted that when the flame was applied from above a yellowish green glow occurred in those parts of the surface of the oxide which were within certain zones of the flame.

When the flame was applied underneath the gauze, heating it to about the same temperature no glow appeared on the upper surface but only on such portions of the oxide as protruded through the mesh and were *directly bathed by the flame*. When the hydrogen jet was below but the gas was ignited only above the gauze the glow appeared in a ring shaped zone at the outer base of the flame. When the flame struck through the gauze or was purposely ignited below the gauze the glow almost or entirely disappeared on both the upper and lower sides of the gauze.

By repeating these observations with such modifications of material and method as suggested themselves we were soon able to establish the following facts:

1. *The effect is not an ordinary temperature radiation*: it exists only in a comparatively narrow range of temperatures within which it rises, reaches a maximum and disappears.

2. It sometimes greatly exceeds in intensity the black body radiation for the same temperature.

3. It is not producible by heating to the required temperature either electrically, as on a heated strip of metal, nor by radiation or conduction, as in a furnace. When a flame is used the glow occurs within the flame only.

4. In the case of photo-luminescent substances the lower limit of temperature of flame excitation is often, but not always, above the point at which ordinary photo-excitation ceases.

5. The active range in general extends above that at which thermo-luminescence occurs.

6. *Not all flames are effective*; in fact the hydrogen flame—and to a

lesser degree flames containing hydrogen such as the colorless flame from ordinary illuminating gas—is so far as known the sole source of this type of excitation. Flames from alcohol, ether and carbon disulphide were found to be *in-active.*

7. The effective region may be described as the outer portion of the reducing zone of the flame. This zone may be readily and very strikingly located and observed by allowing the flame to impinge upon a red hot copper surface. The coating of oxide on the copper is then reduced to metallic form throughout a perfectly defined and sharply bounded region within the flame; the reduced area being distinguished by the marked change in reflecting power and emissivity. The identification of the region of activity with the edge of this reducing zone is therefore very simple.

Temperatures were estimated in these experiments by placing the substance in the form of a fine powder on one of the end faces of a massive cylindrical block of copper about 30 mm. in diameter and 20 mm. thick. A narrow slot in this face received a platinum wire, the copper being hammered down so as to grasp the wire and make it a part of the surface. This copper-platinum contact formed one junction of a thermo-electric element by means of which the temperature of the face of the block could be measured. As we have been unable to find data giving directly in millivolts the E.M.F. of such a copper-platinum junction we insert here for the benefit of those who may have occasion to use this very convenient means of measuring temperatures to 1000° C.,[1] the results of our calibration.

TABLE I.

Millivolts and Temperature Differences of Copper-Platinum Thermo-element.

M.V. = millivolts: t = temperature of hot junction in degrees centigrade—the cold junction being at 0° C.

t.	M.V.	t.	M.V.	t.	M.V.	t.	M.V.
25° C.	0.210	275° C.	2.640	525° C.	7.180	775° C.	14.440
50°	0.425	300°	2.850	550°	7.780	800°	15.075
75°	0.640	325°	3.330	575°	8.400	825°	15.770
100°	0.855	350°	3.750	600°	9.020	850°	16.490
125°	1.069	375°	4.170	625°	9.900	875°	17.320
150°	1.282	400°	4.610	650°	10.820	900°	18.220
175°	1.497	425°	5.060	675°	11.640	925°	18.580
200°	1.707	450°	5.600	700°	12.410	950°	18.940
225°	1.916	475°	6.110	725°	13.120	975°	19.360
250°	2.235	500°	6.076	750°	13.820	1000°	19.760

To ensure snugness of contact between the luminescent substance and the copper, the face of the block was variously grooved and slotted and

in some instances the powder was applied by smoking the block in fumes of a burning metal or over an electric arc, into which the oxide or some salt of the substance under investigation had been introduced. Above 550° temperatures could likewise be determined with the Morse thermo-gauge using a calibration of that pyrometer, previously made, which agreed perfectly with the indications of the thermo-element. The copper block was bored through axially and the hole, one centimeter in diameter, afforded a surface which could be coated and which gave a fair approximation to black body conditions. It was sometimes advantageous to compare the appearance of coatings thus enclosed with those on the open end of the block.

Much attention was given to the difficult question of the difference of temperature between the upper surface of the coatings and the surface of the metal on which they were mounted. Obviously the face of a poorly conducting layer would be cooler than the hot block when out of the hydrogen flame and hotter than the underlying metal when in the flame and it was important to estimate this difference or to know that it could be neglected.

That the difference was small in most cases where the precise temperature was in question we were able to show in many ways.

If for example a coating of one of the sulphides which glows strongly with a green or blue color at a temperature just below the red heat be scattered loosely and with varying thicknesses on the brass gauze or the copper block and excited in the manner already described it is easily possible to render the thickest parts of the coating red hot and non-luminescent while thinner regions within the flame, owing to cooling effects of the metal beneath are below the red heat and strongly luminescent. If now the thinner regions of the coating coincide in color and brightness with the very thinnest and therefore with the underlying metal one may feel assured that temperature measurements of the metal surface will apply up to a certain thickness of coating and even estimate approximately at how many hundredths of a millimeter thickness the departure of the temperature of the surface of the metal will become noticeable. It was found that for thicknesses under 1/10 mm. the departure was not of a size to appreciably vitiate our estimates but that at greater thicknesses it rapidly became significant.

In general estimates of temperature were made with the massive copper block, heated by an electric furnace or a Bunsen flame. Its mass and conductivity were such that the small hydrogen flame applied from above had no measurable effect on the surface, *i.e.*, no effect that could be noticed with the Morse gauge. The only difficulty therefore was of

local heating of the upper layers of the powdered coatings. The above mentioned estimate of the permissible thickness of coatings was confirmed by observations of the heating power of the flame on layers of black oxides the radiating power of which rendered it possible to determine their temperatures directly with the pyrometer. Thus we were able to assure ourselves that for the thicknesses of coating used when temperature measurements were involved the temperature of the surface of the copper coincided with sufficient exactness to that of the luminescent coating. The most satisfactory direct check, on the whole, consisted in substituting for the luminescent powder a coating resembling it as nearly as possible in all other physical characteristics and to observe its appearance in and out of the flame and different zones of the flame. Since under the conditions of our experiments the brightness of these regions was appreciably the same it seemed altogether reasonable to assume that the luminescent surfaces in the study of which we were engaged had approximately the same temperature as the copper block or metal gauze upon which they rested.

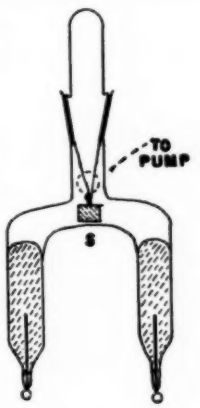

Fig. 1.

This conclusion was indirectly verified in subsequent studies of kathodo-luminescence at high temperatures in which it was found that the temperatures at which calcium oxide and other substances ceased to be active under kathode bombardment were the same as the extinction temperature estimated in the present investigation.

Experiments with Electrostatic Fields.—Deeming it highly probable that the effect in question involved ionization or the production and movement of charged particles, electrostatic fields were brought to bear with lines of force transverse to the stream lines of the flame and also parallel to the same. Although potential differences of many kilovolts were thus applied *no result whatever could be observed.*

Photo-excitation at High Temperatures.—To test as thoroughly as possible the suggestion that flame excitation may be due to ultra-violet radiation within the flame itself very powerful photo-excitation was applied to certain of our substances at high temperatures. To this end a mercury arc lamp was made of the form shown in Fig. 1. Above the arch of the inverted U-tube which contained the arc a vertical tube *T* was inserted. This had a glass stopper to which a body (*S*), to be subjected to radiation from the arc, could be suspended so as to hang within the path of the discharge. Various substances which were excited by the

flame were thus treated; *e.g.*, calcium oxide, zirconium oxide, etc. These remained completely inert whereas a synthetic ruby under like conditions was exceedingly brilliant until it reached its temperature of extinction when it was suddenly quenched. It seemed highly improbable that any photo-excitation by the hydrogen flame could approach that of this mercury arc in intensity.

Combustion a Necessary Factor.—This fact is very prettily brought out in an experiment based upon a method described by Simonini[1] and used by him in the study of the luminous properties of gas mantles.

A glass tube about 1 cm. bore and 20 cm. in length was coupled to the nozzle of an oxy-hydrogen blast-lamp as in Fig. 2. A small amount of

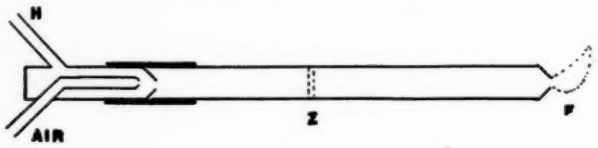

Fig. 2.

calcium oxide was distributed within the tube from end to end, coating the inner walls thinly.

When hydrogen was run through the tube and ignited at the free end (*F*) there was no excitation of the calcium oxide even though the tube was heated to a dull red. The addition of air admitted through the oxygen intake likewise produced no effect, whatever the temperature, until the supply of oxygen within the tube caused the flame to "strike back." When this occurred the progress of the flame through the tube could be traced by the luminescence of the oxide.

By adjustment of the supply of oxygen the zone of combustion (*Z*) could be brought to rest within the tube and its position was then clearly marked by a narrow ring of luminescent powder.

From these observations it appears:

1. That luminescence is not produced by the mere presence of hydrogen, at temperatures up to the red heat.

2. That hydrogen and oxygen mingled as above have no effect.

3. That where combustion occurs luminescence is produced but then only in the narrow zone where the reaction is in progress.

In the hydrogen flame we have, besides the combination with oxygen, the production of H_1 or H_3 or both as has been shown by Langmuir,[2] by Wendt[3] and others.

[1] A. Simonini, Trans. Illuminating Eng. Soc., 1909, p. 648.

[2] Langmuir.

[3] Wendt.

It might reasonably be suspected, that disturbances incident to return of these unstable forms to H_2 would produce the phenomenon of luminescence in substances with which the gas was in contact. If so the atomic hydrogen would presumably be swept along with the flow of gas in the experiment just described and the luminescent zone extended, with decreasing intensity for some distance from the boundary between oxygen and hydrogen within the tube. That, however, was not the case and it was found by numerous modifications of the experiment that the active zone was equally sharp and well defined whether combustion occurred by igniting a jet of hydrogen in oxygen or vice versa. The luminescence always occurs however *within the hydrogen* next to the boundary between hydrogen and oxygen.

Experiments with hydrogen filled tubes at various pressures, within which it was attempted to produce atomic hydrogen by powerful electric discharges both direct and alternating and to expose our luminescent oxides to this gas when heated to a wide range of temperatures, were all equally without result.

The evidence at present therefore appears to be against the assumption that the presence of H_1 or H_3 in contact with the luminescent materials is the cause of the effect which we have called flame excitation.

Active and Inactive Materials.

Some of the materials sensitive to flame excitation, in addition to the old lime already mentioned are:

(*A*) Calcium oxide, from various other sources, zirconium oxide, magnesium oxide, silicon oxide, aluminum oxide, zinc oxide.

(*B*) Certain phosphorescent sulphides such as Sidot blende and Balmain's paint and sulphides of the Lenard and Klatt type.

Not all the Lenard and Klatt preparations are active. Out of forty-four tested the twelve given in the following table responded strongly; a few others dimly.

(*C*) In addition to the above, white saphires, corundums, synthetic rubies and crystals of calcite, fluorite, kunzite, and topaz responded to the flame excitation; as did boric acid, telluric acid, and very pure samples of cadmium phosphate, calcium carbonate, calcium sulphide and of zinc sulphide all incapable of photo-excitation. On the other hand all dark-colored substances, tried, such as the oxides of copper, lead, iron, nickel, manganese, cobalt, cadmium, tungsten., etc., were inert. So likewise were the oxides of bismuth, antimony, barium and strontium; samples of the oxides of erbium, cerium, lanthanum, and thorium; materials from a freshly ignited Welsbach mantle and from a Nernst filament and specimens of uranium glass, didymium glass, fused quartz, etc.

TABLE II.

List of Lenard and Klatt Sulphides that Responded Strongly to Flame Excitation.

Compound.	Excitation.	Color and Brightness.		Spectrum.
No. 3 CaS, Bi, Na_4BO_4	Photo- Thermo- Flame-	Blue-violet Deep blue Blue-white	Very bright Very bright Very bright	Green to violet
No. 5 CaS, Mn, NaF	Photo- Thermo- Flame-	Yellow Absent Yellow-green	Dim —— Very bright	Red to green (no violet)
No. 9 SrS, Bi, K_2CO_3	Photo- Thermo- Flame-	Blue-green Very dim Blue-green	Bright —— Bright	Green to blue (no red)
No. 27 CaS, Sb, Na_2F_3	Photo- Thermo- Flame-	Blue-green Dim Blue-green	Medium —— Very bright	Green to blue (no red)
No. 29 CaS, Pb, NaF	Photo- Thermo- Flame-	Green Absent Pale green	Very dim Absent Very bright	Green to blue (no red)
No. 32 CaS, Cu, LiF	Photo- Thermo- Flame-	Blue —— Blue-green	Bright Absent Medium	
No. 34 CaS, SrS, Bi, NaF	Photo- Thermo- Flame-	Blue —— Deep violet	Bright Absent Moderate	
No. 36 CaS, SrS, Bi, Na	Photo- Thermo- Flame-	Blue —— Blue-green	Medium Very dim Bright	Blue to green (no red)
No. 38 CaS, SrS, Bi, Na	Photo- Thermo- Flame-	Blue —— Blue-green	Medium Absent Bright	
No. 39 CaS, SrS, BaS, Cu, Bi, LiF	Photo- Thermo- Flame-	Yellow-green —— Pale yellow	Medium Absent Medium	
No. 43 CaS, Bi, Cu, CaF	Photo- Thermo- Flame-	Blue-green Blue-green Blue-white	Bright Very bright Bright	
No. 44 CaS, Bi, Sb, Na	Photo- Thermo- Flame-	Blue —— Pale green	Medium Absent Very bright	Blue to green (no red)

Calcium tungstate, one of the most brilliant of fluorescent substances under the action of X rays is quite inert when subjected to the hydrogen flame as is artificial willemite, which is perhaps the brightest of photo-luminescent bodies.

THE ACTIVE OXIDES.

Calcium oxide whether obtained from the slaked lime mentioned above or from calcite crystals or by burning metallic calcium gave the same strong yellow glow in the hydrogen flame and the various preparations were active through the same range of temperature. Complete extinction occurred for all these samples at about 690° C.[1] Since the oxide from these varied sources was equally and similarly active it does not seem probable that the effect depends on the presence of small amounts of an accidental impurity as has been shown[2] to be the case in the photo-luminescence of Franklin Furnace calcites.

Such admixtures however may modify the character of the glow. Thus a specimen of calcium oxide to which a trace of a bismuth salt had been added, with subsequent heat treatment, showed a brilliant *blue-green* fluorescence instead of the usual *yellow*, when excited by the hydrogen flame.

Of *zirconium oxide* the preparations available were some old disks from a Linnemann[3] "zircon-light" apparatus[4] and a purified white oxide in powdered form from the Cornell Department of Chemistry.

In the hydrogen flame the Linnemann disks gave a remarkable deep red and in cooler portions a pale blue-green fluorescence. The white powder showed the blue-green only.

The red fluorescence is probably due to some impurity but apparently not to the boric acid used as a binder. At least admixture of the latter with the pure oxide failed to impart even a suggestion of a ruddy tone to the glow in flame or in vacuo.

To estimate the active range of luminescence, part of a disk was pulverized and strewn on the surface of the cold copper block in the usual manner.

The observed phenomena were as follows:

1. Pale blue green fluorescence which began very definitely when the block was heated to 76° C. It remained visible until the upper limit of temperature was reached at 372°. Throughout this entire range of

[1] To be more precise it should be stated that the luminescence spectrum consists of two or more overlapping bands. One of these, lying chiefly in the red-yellow, begins to respond to flame excitation at a lower temperature than the other and vanishes at about 670°. The other band which is yellow-green becomes visible only at higher temperatures and does not disappear altogether until the oxide reaches a temperature of 725°. Details of the experiments on which this statement is based will be given in forthcoming paper.

[2] Nichols, Howes and Wilber, PHYSICAL REVIEW (2), XII., p. 35 (1918).

[3] Linnemann, Dingler's Journal, CCLX., p. 218

[4] The disks according to Erdmann (Chimie, p. 139) were prepared by moistening zirconium oxide—"Zirkonerde"—with a solution of boric acid, compressing and baking. Such disks were used by Rubens and others as sources of infra-red radiation.

temperatures there was no suggestion of the red luminescence but on heating to a dull red we saw:

2. The deep red luminescence already described.

Lower limit	440°
Upper limit	720°

On account of the ruddy color there was some difficulty in distinguishing with certainty between this luminescence and ordinary red heat, but its temperatures of appearance and vanishing were quite fixed and definite and the color was quite different from that of a red hot body, being more nearly of the approximately monochromatic tint which one gets through ruby glass.

Silicon oxide whether in the form of a precipitated powder, or a smoked coating sublimed in the arc from metallic silicon or from quartz exhibits a pale green-white, nearly white fluorescence in the hydrogen flame. Quartz crystal, not crushed, is difficult of excitation and fused quartz appears to be inactive.

To determine the temperature range a very thin coating of the precipitated silica was rubbed into the surface of the copper block. Inactive at room temperature, the fluorescence came very suddenly into view on heating the block. The threshhold was very sharp and definite; so much so that a drop of cool water applied to the surface of the block two centimeters away would immediately extinguish the luminescence within the flame on the side nearer the drop by the slight local fall in temperature thus temporarily produced. The range of temperatures was found to be

Lower limit	85°
Upper limit	367°

Aluminum oxide in the form of a fine powder from the department stock room was notable for the very low temperature at which it began to glow. The temperature range for the pale green fluorescence was:

Lower limit	52°
Upper limit	692°

At higher temperatures this coating when in the active zone of the flame presented a curiously mottled appearance there being patches of reddish hue others green-blue and others nearly white. This strange mosaic might have been ascribed to a lack of homogeneity in the material used but for the fact that another coating obtained from the smoke of burning aluminum was similarly variegated under kathode bombardment. It seems to be a general property of this oxide and is under detailed investigation.

Aluminum oxide was found to be susceptible to flame excitation like-

wise in crystal form. A synthetic ruby gave a greenish glow without a trace of the well-known deep red photo- and kathodo-luminescence. A white sapphire known to be synthetic fluoresced a very delicate pale green in the active zone of the flame; although it like the ruby glowed red under kathodo excitation.

Magnesium oxide obtained by smoking the copper block in the fumes of burning magnesium wire showed a fine white flame fluorescence:

Lower limit	76°
Upper limit	680°

Magnesium carbonate, the ordinary compressed pharmaceutical preparation, showed a similar white fluorescence and since the limits were practically the same as for the oxide (76° to 670°) it may well be that there is partial decomposition of the carbonate to oxide as in the case of the calcite referred to in a subsequent paragraph.

At temperatures much higher than the above, blended with the brilliant white selective radiation from magnesium oxide there is another stage of luminescence, blue-violet in color, the exact temperature relations of which are difficult and have not yet been determined.

Zinc oxide like the oxides of zirconium and magnesium exhibits two stages of flame luminescence but in this case the lower stage is somewhat difficult to bring out. The color of the luminescence is an intense red. It begins at 568° and above 700° [704°] it goes over into a higher stage characterized by a blue-green glow which persists to 948°. The total range for both bands is thus: Lower limit, 568°, upper limit, 948°. The green glow is brilliant and so strikingly different in hue from the temperature radiation on which it is superimposed, that it admits of very definite quantitative study.

An account of these measurements and of observations on the more difficult higher stage luminescence of magnesium oxide are to be presented in a separate paper.

The Phosphorescent Sulphides.

These substances differ from the oxides just considered in that they are strongly photo-luminescent at ordinary temperatures owing to the artificial or sometimes accidental admixture of some active element, such as, bismuth, copper, lead, or antimony.

The solid solvent of those which respond to the flame is usually, calcium sulphide; sometimes strontium sulphide; never, so far as at present known, barium sulphide. Zinc sulphide in the form of Sidot blende is also highly sensitive to flame excitation.

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An account of these measurements and of observations on the more difficult higher stage luminescence of magnesium oxide are to be presented in a separate paper.

THE PHOSPHORESCENT SULPHIDES.

These substances differ from the oxides just considered in that they are strongly photo-luminescent at ordinary temperatures owing to the artificial or sometimes accidental admixture of some active element, such as, bismuth, copper, lead, or antimony.

The solid solvent of those which respond to the flame is usually, calcium sulphide; sometimes strontium sulphide; never, so far as at present known, barium sulphide. Zinc sulphide in the form of Sidot blende is also highly sensitive to flame excitation.

A phosphorescent sulphide differs from the oxides already described in

that at room temperature, it usually reacts to the ultra-violet radiation from the hydrogen flame even at a considerable distance.

When warmed by the radiation, even though not in contact with the flame, it shows thermo-luminescence. Then, having become dead to all photo-excitation it will still respond to flame excitation if brought into the active zone and heated there to the proper temperature.

That the flame excitation of these sulphides is not due to a conversion of the surface layer into the oxide is indicated by the color of the luminescence which corresponds to the kathodo- or photo-luminescence of the sulphide and not to the flame effect as observed in the oxide.

Spectrophotometric measurements, moreover, have been made by Dr. H. L. Howes. These were extended to three Lenard and Klatt sulphides the spectra of which had been previously studied[1] and it was found that the spectra of these, under flame excitation, were composites of the same groups of narrow components which make up their spectra in the case of photo- or kathodo-luminescence.

The response of these substances to light after cooling indicated that no permanent decomposition occurred under the action of the flame. At higher temperatures, as in the electric arc, the sulphides were, however, converted into the corresponding oxide and the flame luminescence became that characteristic of the oxide.

While a so-called active element in solid solution is generally conceded to be necessary to the production of a phosphorescent sulphide, this does not seem to be essential in flame excitation. What was found for the oxides in this respect, applied to the sulphides also. A sample of zinc sulphide which was so pure as to be quite inert under the iron spark, both as regards fluorescence and phosphorescence, responded brilliantly with a yellow-green glow in the hydrogen flame at a red heat. The effect became distinctly visible at 560° and was strong at 616°; much brighter indeed than the luminescence under like conditions of specimens of Sidot blende especially prepared with a view to phosphorescent properties. This same pure zinc sulphide responded strongly under kathode rays at room temperature but was without persistent afterglow.

Attempts to determine the upper limit of the luminescence were less definitely successful. Above 900° the glow was feeble but discernible and at 934°, the highest temperature attained in this run, it had not altogether vanished. We were now almost at the upper limit for ZnO as described in a previous paragraph and it seemed almost certain that portions of the sulphide had been converted into the oxide.

Calcium sulphide, when pure, is inert alike under photo-excitation

[1] Nichols, Proc. of Am. Philosophical Society, LV., p. 496 (1916).

and kathode bombardment. A sample which fulfilled these criteria of freedom from admixtures was found however to be excited by the hydrogen flame through an unusually wide range of temperatures. When placed in a very thin layer on the surface of the copper block and gently heated it showed flame luminescence quite definitely when the block was heated to 60° C. The color was *bluish-green*. The intensity increased in brightness with rise of temperature and below the red heat there was a rather sudden change to a much brighter yellow glow—a lemon yellow and very luminous. This continued with growing and then waning intensity until a cherry red heat of the block, when it vanished. Measurements of the highest temperature at which the luminescence could just be perceived were made with the Pt-Cu thermo-junction, and nearly simultaneously with the Morse thermo-gage, with the following result:

Vanishing temperature by pyrometer	744° C.
Vanishing temperature by thermojunction	735° C.

An attempt to pick up this temperature again as the block slowly cooled gave

Pyrometer	731°
Thermojunction	730°

Taking the average of these four estimates we have 737° C. as the temperature of the surface of the block and the approximate temperature of the coating. The yellow glow increased rapidly as the block cooled further and was apparently at its brightest somewhat below 500°. The transformation to the blue-green occurred at 330° according to the indications of the thermo-junction. At this temperature the very thinnest parts of the coating which were nearest the metal and therefore cooler had changed while the high points were still a bright yellow.

Calcium sulphide may therefore be added to the list of those substances having two stages of flame luminescence. The temperature ranges were as follows:

Lower stage	60° to 330° (blue-green).
Upper stage	330° to 737° (bright yellow).

Calcite, Fluorite and other Crystals.

Of the crystals thus far tested for flame luminescence calcite and fluorite especially merit detailed mention.

A crystal of calcite in the flame is at first inactive. Then its edges begin to glow brightly, framing the otherwise dark crystal in lines of light, and the light very gradually extends over the faces. Examination shows that decomposition of the carbonate precedes luminescence. This

occurs first along the exposed edges and later over the faces of the crystal. The glow seems to depend on the presence of this film of oxide.

The obviously localized and superficial glow produced by flame excitation is altogether different from the phenomenon of thermo-luminescence, as produced by the same flame, when applied to certain calcites or to kunzite or fluorite. In thermo-luminescence the effect is seen to pervade the entire body of those portions of the crystal which have attained the requisite temperature; in flame excitation proper it is confined to those portions of the *surface* which are within the active zone.

Several specimens of fluorite were examined. Of these some showed the characteristic blue-violet fluorescence under the action of the iron spark. Others, which were mostly of a yellowish body color did not respond visibly to photo-excitation. All however were more or less thermo-luminescent and all showed the same *pale greenish white* flame-luminescence.

The temperature limits observed in the case of one of the almost white, clear, transparent crystals were as follows:

Lower limit	58° C.
Upper limit	310° C.

Upon heating to a much higher temperature this crystal began to be converted into the oxide along its edges. Upon cooling it developed a very bright fluorescence under excitation of the flame having the yellow color characteristic of calcium oxide, which was picked up at about the usual temperature, *i.e.*, 690°.

The significant feature in the behavior of these fluorites is that while they differ in color and in their response to photo-excitation—differences usually ascribed to the presence or absence of admixtures of the rare earths, etc.—they are all much alike, if not identical under flame excitation; from which, as in various previous experiments, we infer that flame excitation does not depend on foreign admixtures.

A similar result is obtained in the case of aluminum oxide when crystallized either as synthetic ruby or white synthetic sapphire, or as native sapphire from North Carolina, or as corundum. All of these crystals showed the same *pale green* fluorescence as the powdered oxide described in the paragraph on the active oxides. *These crystallized forms* however all gave the red fluorescence of the ruby when tested in the kathode tube because presumably of the presence in all of traces of chromium.

How general an effect flame-luminescence is among crystals and also among compounds other than the oxides and sulphides remains to be determined. As has already been indicated in the list of substances thus far found active, boric acid and telluric acid respond to the flame. Crys-

tals of the latter compound even during the process of being decomposed by the flame glow with a vivid green light.

From our present evidence it appears probable that only those materials will become luminescent under the action of the hydrogen flame which are capable of the proper sort of rapid reduction and subsequent oxidation already described and that the active range of temperatures in each case is that within which the above reaction occurs.

Summary and Conclusions.

1. Flame excitation occurs when certain substances are brought into the zone of a hydrogen flame between the regions of reduction and oxidation.

2. It occurs, so far as known, in certain oxides and sulphides; in a few phosphorescent preparations and in certain crystals and inorganic salts.

3. Many substances strongly luminescent under the action of light, such as calcium tungstate, willemite and many of the Lenard and Klatt sulphides do not respond to flame excitation.

4. The effect does not appear to depend on the presence of traces of some activating elements. Such admixtures may modify the character of the fluorescence; they often inhibit it altogether. Thus pure cadmium phosphate responds to flame excitation, but the addition of manganese, which renders it strongly photo-luminescent with a well-known red phosphorescence, destroys its susceptibility to excitation by the flame.

5. The spectra obtained by flame excitation are characteristic fluorescence spectra consisting of two or more overlapping bands. Each band is made of numerous components forming series of equal frequency intervals and these components in all the cases thus far investigated are identical with the components of the fluorescence spectrum of the substance when photo-excited.

6. Flame excitation occurs through a definite range of temperatures. The temperature limits thus far studied are as follows:

Table III.

Substance.	Lower Limit.	Upper Limit.	Color of Band.	Substance.	Lower Limit.	Upper Limit.	Color of Band.
SiO_2.....	85°	367°	White	ZnO.....	568°	704°	Red
MgO....	75°	680°	Blue-green		704°	948°	Green
Al_2O_3....	55°	692°	Pale green	ZnS.....	0° ±	123°	Yellow
CaO.....	0° ±	600°	Red		211°	575°	Blue-green
	40°	725°	Green	CaS.....	60°	330°	Blue-green
ZrO_2....	76°	372°	Blue-green	CaF.....	58°	310°	Yellow-green
	440°	720°	Red	$Cd(PO_4)_2$..	50°	733°	Yellow

These values are to be regarded as only approximate. The fluorescence in all cases is doubtless complex, there being at least two broad bands. These bands have their own independent ranges of temperature and the effect observed is usually a composite. In the case of CaO the two temperature ranges greatly overlap. In that of ZnO they barely overlap. The bands of ZrO_2 and ZnS are quite distinct as to temperature. The phenomenon with the sulphides is complicated by the probability of a change into oxides at the higher temperatures. Both CaS and ZnS exhibit bands above the temperatures of the table which coincide as to color and upper limit with those of the corresponding oxide. These have not been included.

7. Excitation of these substances to fluorescence does not occur by heating in air or hydrogen nor in vacuo nor by contact with a jet of hydrogen while hot unless the hydrogen be ignited and conditions are favorable for free oxidation.

8. Luminescence does however occur by kathode bombardment throughout essentially the range of temperatures indicated under section (6) but the effect is not identical since the bands are not excited in the same proportions as in the flame. X rays also excite many of the substances in question at high temperatures.

9. The after glow, in so far as the same has been studied follows the law of vanishing phosphorescence. Details will appear in a subsequent paper.

10. The action of strong static fields having been found without effect; also the use of powerful photo-excitation at high temperatures and the substitution of other flames for hydrogen, the explanation put forth in an early paragraph is tentatively adopted, *i.e.*, that flame excitation is a dual process in which reduction and oxidation rapidly succeed each other in an unstable layer within the flame between the region where reduction is continuous on one side and oxidation on the other. The alternative suggestion that it is due to disturbances arising from the formation of H_1 or H_3 and their subsequent recombination to H_2 is not positively excluded.

Numerous experiments and measurements not given in the present paper are already completed and ready for publication and others are in progress.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
October, 1920.

THE SPECTRAL STRUCTURE OF THE LUMINESCENCE EXCITED BY THE HYDROGEN FLAME.

By Horace L. Howes.

Synopsis.

Luminescence Spectra Excited by the Hydrogen Flames.—Calcium oxide and the Lenard and Klatt phosphorescent sulphides Nos. 3 and 13 were studied by the aid of a spectrophotometer. The broad-banded spectra were found to consist of the same components, individually overlapping and forming overlapping series of constant frequency intervals. Series and individual components were identified with those entering into the spectral structure of these substances when excited to fluorescence by light, but the relative intensities of the bands were found to differ with the mode of excitation.

THE discovery of the luminescence of many substances when partially bathed in the hydrogen flame but at a temperature below red heat has very recently been reported by Dr. E. L. Nichols.[1] Since the hydrogen flame luminescence was found to be very brilliant when air-slaked lime was presented to the outer layer of the flame this substance was first studied by the author. The specimen was found to fatigue or disintegrate under the flame hence a fine brass gauze was filled with the lime and slowly rotated at a constant speed before the flame. The size of the flame was kept as constant as possible by the use of a reducing valve and a gas regulator. The impossibility of maintaining the flame sufficiently constant necessitated the use of the statistical method and each spectrophotometric curve represents an average of many trials, properly weighted. Fig. 1 represents the distribution of brightness in the visible spectrum of the air-slaked lime. The luminescence before dispersion was of an orange-yellow color and the distribution of color between wave-lengths of .61 μ and .53 μ is indicated in the curve, the maximum lying near .56 μ. The subsidiary crests and shoulders, marked by short vertical lines, although of widely varying intensities and forms are not without a systematic arrangement. When the reciprocals of the wave-lengths of the crests are studied they are readily found to lie in two series of a constant interval of 42 units, designated as the "*A*" or "*B*" series. These wave-lengths and reciprocals are given in Table I. together with the wave-lengths obtained from the luminescence by excitation with the iron spark of natural calcite. The spectral analysis

[1] E. L. Nichols, 1920.

by the iron spark excitation was done by Nichols, Howes and Wilber with the aid of an elaborate photometric control.[1] The positions of the bands are the same under the two different modes of excitation and the structure of the calcite and air-slaked lime must contain the same lumin-

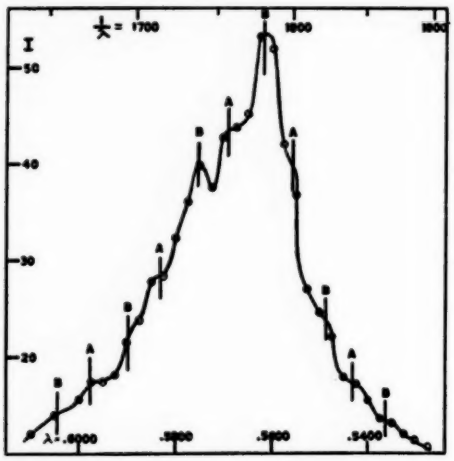

Fig. 1.

escing system. Because of the nature of the crests it is not considered possible to locate the positions of the bands within ten Ångström units of the proper places hence the agreement found in Table I. is conclusive. The more interesting fact is that although the general form of the spectral

TABLE I.

Luminescence of Lime.

Series *A* (Interval = 42).				Series *B* (Interval = 42).			
Band *A*.	Hydrogen Flame, λ.	Iron Spark, λ.	Frequency Unit, $1/\lambda \times 10^3$.	Band *B*.	Hydrogen Flame, λ.	Iron Spark, λ.	Frequency Unit, $1/\lambda \times 10^3$.
A_4.....	.5975	.5977	1673	B_4.....	.6045	.6046	1654
A_5.....	.5833	.5831	1715	B_5.....	.5894	.5896	1696
A_6.....	.5689	.5692	1757	B_6.....	.5755	.5754	1738
A_7.....	.5563	.5559	1799	B_7.....	.5617	.5618	1780
A_8.....	.5432	.5432	1841	B_8.....	.5488	.5488	1822
				B_9.....	.5365	.5365	1864

envelope is quite different from that obtained by the iron spark excitation yet the partially resolved crests are in no way shifted by change in mode of excitation. The form of the envelope indicated in Fig. 1 corresponds

[1] Nichols, Howes and Wilber, "The Photo-luminescence and Kathodo-Luminescence of Calcite," PHYSICAL REVIEW (2), Vol. 12, p. 365; Nov., 1918.

more nearly to that for the calcite under the iron spark excitation at 200° C. than at 20° C. See Fig. 13, page 364; Nichols, Howes & Wilber (loc. cit.). This is naturally so, since the hydrogen flame warmed the specimen to a temperature of approximately 350° C. Temperature changes the relative intensities of the crests but the "*A*" and "*B*" series are independent of both temperature and mode of excitation for their frequencies.

The Lenard and Klatt sulphide known as No. 3 is a calcium compound containing both bismuth and fluorine as active elements. Since the spectral structure of this preparation had previously[1] been studied under the excitation of the zinc spark it was thought that an interesting comparison could be based on the resemblance between the zinc spark and hydrogen flame excitation. The spectrum had been known to exhibit a strong band in the blue and a broader but much dimmer band in the orange. See Fig. 4, p. 265, E. L. Nichols (loc. cit.). Under hydrogen flame excitation at a very moderate temperature the blue band was found in approximately the same location but the orange band was of much greater proportionate intensity. To produce the orange band the mounting was rotated more slowly to allow a more favorable temperature. In some trials the specimen was mounted in the gauze without rotating

TABLE II.

Luminescence of Calcium Bismuth Sulphide, No. 3.

Hydrogen Flame, λ.	Zinc Spark, λ.	Frequency Unit, $1/\lambda \times 10^3$.	Frequency Interval, $\Delta(1/\lambda) \times 10^3$.
.6365	——	1575	39
.6208	.6200	1614	39
.6048	.6049	1653	39
.5912	.5910	1692	39
.5785	.5781	1731	39
.5650	.5650	1770	39
.5530	.5528	1809	39
.5415	.5411	1848	39
.5300	.5300	1887	

but in such cases every precaution was taken to maintain the temperature constant but below red heat. A comparison of the location of the crests of the complex orange band under the two different modes of excitation is given in Table II. One new band was found at .6360 μ. The agreement is as good as could be expected and no systematic deviation is evident. The bands are undoubtedly in the same position and

[1] E. L. Nichols, "Spectral Structure of the Phosphorescence of Certain Sulphides," Proceedings of the American Philosophical Society, Vol. 55, p. 496, 1916

are spaced by a constant frequency interval of 39 units. Again the spectrophotometric curve is not similar to that obtained by the zinc spark excitation, as can be noted by a comparison of Fig. 4, page 265 of the paper last cited with Fig. 2 of the present paper. Evidently a similar

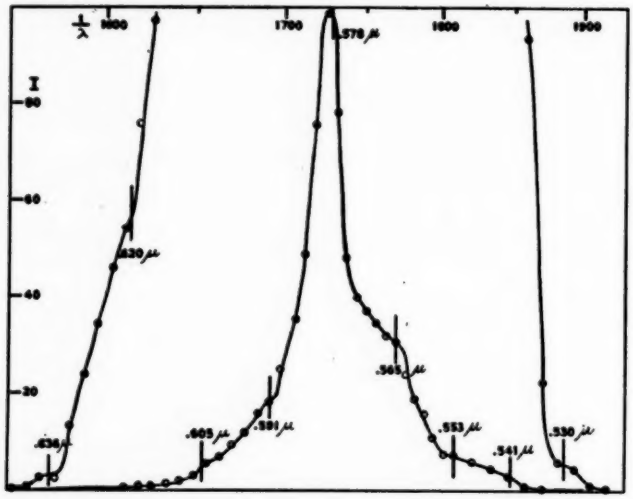

Fig. 2.

vibrating system has been excited by the new method at a slightly higher temperature since the same bands, having the same constant frequency interval of 39 units, are present.

To make sure that the identity of band structure was not accidental another sulphide of Lenard and Klatt containing strontium and bismuth was excited by the hydrogen flame. This compound, known as No. 13, had been previously studied by Nichols.[1] The undispersed light was of a delicate blue color and so dim that a considerable study of the method of mounting was necessary before sufficient radiation could be obtained to render spectrophotometric measurements possible. The light from the 4-candle-power tungsten lamp was reduced greatly by interposing three or more sheets of white porcelain between it and the comparison slit. The spectrophotometric field was almost colorless and the time required for eye accommodation about thirty minutes. Since the range of intensities was unusually great, the curve shown in Fig. 3 is broken into five sections to facilitate the location of the partially resolved crests. With the exception of crest .494 μ and .480 μ the presence of partially hidden bands is conclusive. Table III. gives the wave-lengths of these bands together with those of the same specimen excited by the zinc spark.

[1] E. L. Nichols, loc. cit.

There is again evidence of an identical underlying system of component bands under the two different modes of excitation. The distribution of intensities is again different and two new bands were discovered which fortunately fit the series intervals. The new bands are .5414 μ and .5080 μ.

As a digression from the field of comparison of the hydrogen flame

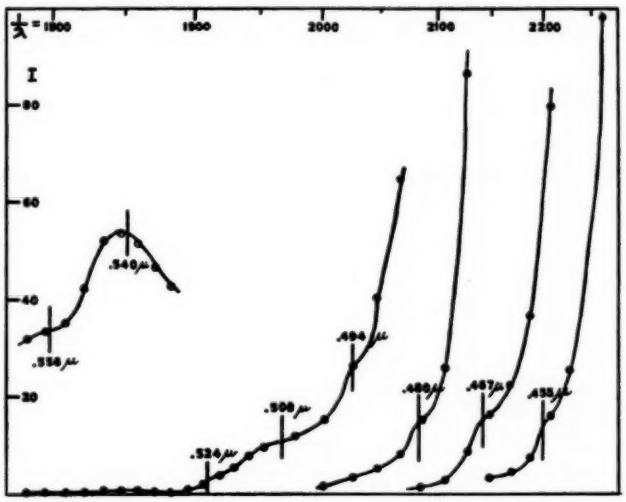

Fig. 3.

excitation with that of the metallic spark an attempt was made to compare two Lenard and Klatt preparations which have the same solid solvents and active metals but a different flux under the same hydrogen flame. The formula given for the Lenard and Klatt sulphide No. 13 is $SrSBiNa_2SO_4$ and the series has been given in Table III. The formula given for the Lenard and Klatt Sulphide No. 9 is SrSBiK Phosphate;

TABLE III.

Luminescence of Strontium Bismuth Sulphide, No. 13.

Hydrogen Flame, λ.	Zinc Spark, λ.	Frequency Unit, $1/\lambda \times 10^3$.	Frequency Interval, $\Delta\ 1/\lambda \times 10^3$.
.5565	.5562	1793	58
.5414	——	1851	58
.5235	.5238	1909	58
.5080	——	1967	58
.4938	.4938	2025	58
.4800	.4801	2083	58
.4668	.4670	2141	
.4546	.4547	2199	

that is, both contain strontium sulphide with bismuth for the element active in luminescence but the flux is a sodium sulphate in the first preparation and a potassium phosphate in the second preparation. The problem was to ascertain the effect of change of flux on the band series. It was thought that the flux might produce little or no effect on the location of the bands. The hydrogen flame was applied to preparation No. 9 and a beautiful dull blue glow appeared. The spectrophotometric curve, not presented in this paper, was obtained with some difficulty from six independent explorations and found to be somewhat similar to that obtained for specimen No. 13. Table IV. gives the wave-lengths and frequency units for comparison with Table III. The interval for No. 9 is 51 frequency units while that for No. 13 is 58 frequency units and it is observed that none of the bands occupy the same positions in

TABLE IV.

Luminescence of Strontium Bismuth Sulphide with Potassium Phosphate Flux, No. 9.

λ	$1/\lambda \times 10^3$.	$\Delta(1/\lambda) \times 10^3$.
.5525	1810	51
.5373	1861	51
.5230	1912	51
.5094	1963	51
.4965	2014	51
.4843	2065	51
.4726	2116	

both spectra. Band .5235 μ of No. 13 and .5230 μ of No. 9 are the most nearly together. The nature of the flux, then, determines in part the disposition of the bands, although the presence of the same base is the predominant factor.

A somewhat parallel phenomenon is observed when the luminescence series of lime (Table I.) is studied in comparison with the series obtained from the Lenard and Klatt sulphide No. 3. (Table II.) Both contain calcium as a base; the active element in the lime is unknown while that in the Lenard and Klatt sulphide is bismuth together with fluorine. The frequency interval in both series found in the spectrum of the lime is of 42 units and that of the calcium bismuth preparation is 39 units. It is evident that the strontium or the calcium when used as the base of the compound largely determines the frequency interval.

PIEZO-ELECTRIC AND ALLIED PHENOMENA IN ROCHELLE SALT.[1]

By J. Valasek.

Synopsis.

Electric Hysteresis in Rochelle Salt.—On the doublet theory of dielectric action, the dielectric displacement D, electric intensity E, and the polarization P are analogous to B, H, and I in the case of magnetism. Rochelle salt shows an electric hysteresis in P analogous to the magnetic hysteresis in the case of iron. The loops obtained are displaced from the origin by an amount which gives a measure of the *permanent polarization in the natural state.* The moment per unit volume in the natural state is of the order of 50 e.s.u./cm.^{2} under ordinary conditions, this being about 13.3×10^{-21} e.s.u./cm.^{2} per molecule.

Piezo-electric Response of Rochelle Salt in an Electric Field.—Mechanical force and electric field are to a certain extent equivalent in producing a change in the piezo-electric moment. Measurements of the piezo-electric response in various applied fields therefore give curves of the same nature as the curves of dD/dE against E. Accordingly they show maxima displaced from the origin by the effect of permanent polarization.

RECENT work on the piezo-electric and allied properties of Rochelle salt has brought to light a number of peculiarities in the electrical reactions of this substance to applied forces and electric fields. A survey of these effects is given by Capt. H. O. Wood in a report dated March, 1919. In the first place, observations by Cady[2] show that the piezo-electric response, or quantity of electricity set free by a pressure is not proportional to the force, as is demanded by the commonly accepted theory of piezo-electricity. Initially the response is more nearly proportional to the square of the force, subsequently increasing less and less rapidly and finally tending toward more or less of a saturation value. Investigating the action of the crystal as the dielectric in a condenser Anderson[3] has found that instead of the usual relation $Q = CV$ between the charge and the applied electromotive force, there exists a more complicated relation much like that described above as holding between piezo-electric response and applied force. The capacity of the condenser and hence also the dielectric constant of Rochelle salt thus appears to be a

[1] Presented at the meeting of the American Physical Society in Washington, Apri 23–24, 1920

[2] W. G. Cady, Report to National Research Council, May, 1918.

[3] J. A. Anderson, Reports to National Research Council, March and April, 1918.

function of the applied electromotive force. Moreover Anderson found that the ballistic galvanometer charge and discharge throws were unequal for fields of one sign but smaller and equal for fields of the opposite sign. In explanation he suggests that an electro-striction effect enters into and modifies the action of the crystal as a condenser. It appears however, that these results can be accounted for by a hysteresis in the crystal similar to ferromagnetic hysteresis, in a manner suggested by Professor W. F. G. Swann, as follows:

On the doublet theory of dielectric action, the dielectric displacement D, electric intensity E, and the polarization P (moment per unit volume) are analogous to B, H, and I in the case of magnetism. They are connected by the relation $D = E + 4\pi P$, and the charge Q of a parallel plate condenser of plate-area S is equal to $Q = S(E + 4\pi P)/4\pi = (S/4\pi)D$. The alteration in the charge density on the plates of the condenser is thus a measure of the alteration in the dielectric displacement D. A hysteresis in P and consequently in D would show itself in measurements of the charge on the condenser. A permanent polarization, such as is demanded by Lord Kelvin's theory, would be made measurable by a displacement of the center of the loop with respect to the origin. When this original polarization is disturbed, a restoring force is brought into play and it appears from experiments on fatigue of Rochelle salt crystals that the restoration of the permanent polarization takes place quite gradually. Suppose then, that the original condition of the crystal is represented by some point on, or near, the upper right-hand part of the hysteresis loop. A field tending to decrease the natural polarization will bring into play a larger hysteresis effect, causing the charge and discharge throws to be unequal to an extent depending on the rapidity with which the restoring force acts. For throws in the opposite direction, however, there will be less of a hysteresis effect and the throws will be more nearly equal. When these conditions were imposed on iron in a magnetization experiment, curves like Anderson's were obtained.

In the apparatus set up to verify this hysteresis effect provisions were made for the regulation of temperature and humidity as well as applied force and electric field. Rochelle salt is often very sensitive to slight changes in the surrounding conditions. The variations in the charge on the electrodes of the crystal were measured by a Leeds and Northrup high-sensitivity ballistic galvanometer. The crystal and all directly connected parts were enclosed in a glass container as shown in Fig. 1. This allowed the apparatus to be evacuated or sealed off as desired in order to preserve a constant atmospheric condition. A solenoid with an iron plunger connected to a glass rod was used to apply pressure to the

crystal, which was mounted on a hard rubber block directly below. The weight of the plunger and the effect of residual magnetism were compensated by a spring and screw attachment which could be adjusted from the outside by means of a ground glass tap. To provide for a uniform application of force, the end of the glass rod was furnished with a universal joint arrangement of hard rubber which came into contact with

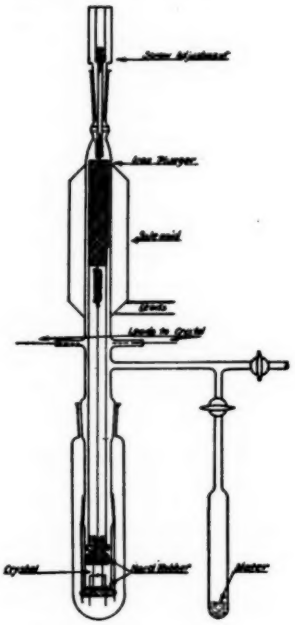

Fig. 1.

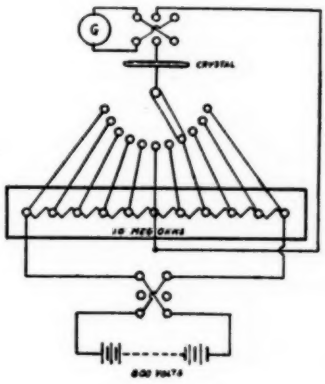

Fig. 2.

the upper end of the crystal plate. The crystal itself was supported on a hard rubber block which was attached by metal strips to an inverted cone. This cone was clamped by a sleeve to a projection on a glass tube as shown in the figure.

For regulating the humidity a bulb containing some water was connected to the glass container. During most of the experiments this bulb was kept packed in ice, giving a constant absolute humidity in the apparatus. The lower portion of the chamber containing the crystal was immersed well into a large Dewar jar which provided heat insulation and also served as means for the regulation of temperature by water baths and freezing mixtures. The temperatures were measured by copper-constantan thermocouples whose junctions were soldered to the tinfoil electrodes on the two faces of the crystal. Two of the thermocouple wires also served as connections to the ballistic galvanometer and to the high voltage potentiometer which regulated the applied electric field.

This consisted of a resistance of ten megohms subdivided into ten equal parts. The various subdivisions were connected to a commutating switch and to the crystal and galvanometer as shown in Fig. 2. The electromotive force usually employed was 800 volts, so that the potential applied was varied from $-$ 400 to $+$ 400 volts in steps of 80 volts each. The potentials used were measured by an electrostatic voltmeter.

The crystal plates were cut from a 100-gram crystal of Rochelle salt which was kindly furnished by Dr. Whitney, of the General Electric Company, to whom the writer wishes to express his appreciation. The plates were cut in the manner described by Cady, namely by a wet thread operating over two pulleys and kept moist by running through some water. The faces were dressed down and smoothened by a moist piece of ground glass. The orientation was such as to give the maximum response. The faces were accordingly perpendicular to the $\bar{a}$ axis of the crystal, the edges making angles of 45° with the $\bar{b}$ and c axes. A compression of the ends of the slab so cut results in a shear in the bc directions. Such a shear produces a maximum of piezo-electric polarization in the direction at right angles to its plane. Denoting the polarization by p_1 and the shear by Y_z we have according to Voigt[1] $p_1 = -\delta_{14} Y_z$. The charge q on a crystal of length l, breadth b, and thickness d, will be: $q = -\delta_{14} F(l/2d)$, where F is the total force on the end of the crystal. The piezo-electric modulus δ_{14} here concerned may have any value between practically zero and about 40×10^{11} e.s.u./dyne. It varies in a capricious manner in response to a number of conditions such as temperature, humidity, and previous history as to electrical and mechanical treatment. These latter fatigue effects, described by Cady,[2] are not permanent but persist often for many hours. The effect of humidity is to make the response greater. Preliminary observations seem to indicate that for changes in temperature the response is small above 35° or 40° C. and that it increases as the temperature is lowered, rising to a maximum at about 0° C. and then slowly decreasing again.

Investigating the behavior of the crystal as a condenser, the conclusions as to a hysteresis in D were verified, the method being that given by Ewing.[3] The hysteresis loops were unsymmetrical as was to be expected if the crystal had a natural polarization. The exact nature of the loops varied with the moisture and temperature conditions. Fig. 3 shows the results of observations taken at a temperature of 23° C. and a relative humidity of 30 per cent. Fig. 4 shows a loop obtained at 0° C., with the

[1] W. Voigt, Lehrbuch der Krystalphysik, p. 818.

[2] W. G. Cady, Report to National Research Council, May, 1918.

[3] Ewing, Magnetic Induction in Iron and other Metals, pp. 358–60.

surfaces of the crystal thoroughly dried. The plate had been kept under cover with P_2O_5 for 15 hours before these observations were made. After 24 more hours under these conditions there was, however, very little hysteresis and only a very small piezo-electric activity, apparently due to the excessive dryness. The moment per unit volume in the natural state varies but is of the order of 50 e.s.u./cm.^{2} under ordinary conditions.

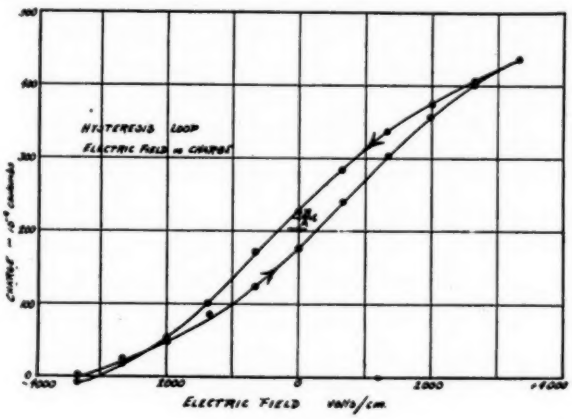

Fig. 3.

It was also observed that the response to pressure underwent a hysteresis with respect to changes in the temperature when the absolute humidity was kept constant by the method described. This hysteresis accordingly may be partly or wholly due to slow changes in the moisture content at the various temperatures concerned. Further work on this point is under way. That such variations in moisture held by the crystal

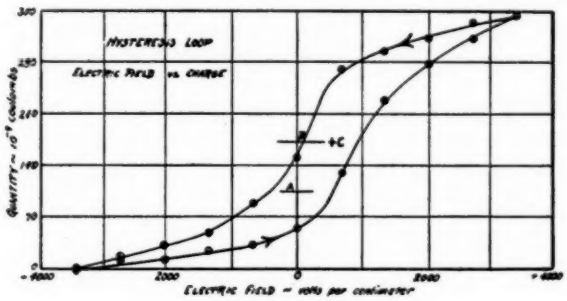

Fig. 4.

actually occur was shown by periodic weighing of two crystal plates which were kept respectively under moist and dry conditions. The changes in weight during the first four days amounted to 5 per cent.

Curves showing the relation between electric field and piezo-electric

response were obtained for a number of values of the applied force (Fig. 5). All of the curves show a maximum for an applied steady field of a certain sign, while they fall off gradually in the opposite direction. These curves may be qualitatively explained on the basis of the hysteresis loops already discussed. It is quite obvious that the change in electric moment, which is the quantity measured, depends on crystalline strain.

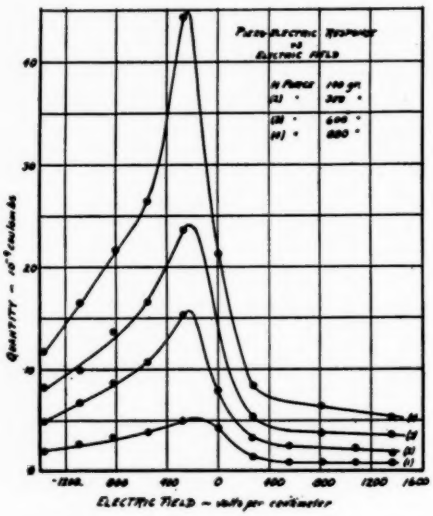

Fig. 5.

Since an electric field produces a piezo-electric stress, one may expect that a mechanical force applied on top of this field is equivalent to a certain extent to a change in applied field. If this equivalence were strictly so, the curves obtained would be just the derivatives of a branch of the hysteresis loop, namely dD/dE versus E. The similarity is apparent, and it is seen moreover, that the curves of Fig. 5 are analogous to the susceptibility curves in the case of magnetism.

Anderson[1] has obtained a set of curves somewhat similar in character by observations made on the charge and discharge of a crystal condenser when under pressure. These results may likewise be attributed to the equivalence of force and electric field in producing a change in the piezo-electric moment. Here, however, the curves are derived from the relation of D and F, which is very much like that of D and E considered above.

The center C of the loop is found by a consideration of symmetry, and may be assumed to represent the condition of no polarization. If the natural condition of polarization is assumed to lie half-way between the two branches of the loop at zero field, then the value of the perma-

[1] J. A. Anderson, Reports to the National Research Council, March and April, 1918.

nent polarization is given by AB, Figs. 3 and 4. Values so obtained give 16 e.s.u./cm.^{2} and 85 e.s.u./cm.^{2} in the two cases respectively. Using 282.2 as the molecular weight of Rochelle salt, 1.76 as its density, and 1.66×10^{-24} grams as the mass of the Hydrogen atom, it follows that there are 3.76×10^{21} molecules per cubic centimeter. The corresponding molecular moments are then 4.26×10^{-21} e.s.u. and 22.6×10^{-21} e.s.u. respectively. If it is assumed that only one electron takes part in the creation of the natural polarization, the moment arm becomes equal to 8.9×10^{-12} cm. and 24.7×10^{-11} cm. in the two cases mentioned above, or at least one thousandth of the diameter of the hydrogen atom. The number of electrons in each molecule may, however, be taken as the sum of the atomic numbers of the elements involved ($NaKC_4H_4O_6 \cdot 6H_2O$) giving a value of 140. These may be equally affected in the natural condition of the crystal, which would indicate a separation of 6.4×10^{-14} cm. and 3.4×10^{-13} cm. respectively, which is the order of the diameter of the electron.

The writer takes this opportunity to thank Professor W. F. G. Swann for his continued interest in this research.

University of Minnesota,
December 4, 1920.

DAMPED ELECTRIC OSCILLATIONS.

BY R. B. ABBOTT.

SYNOPSIS.

Historical.—Logarithmic and linear damping in electric circuits. A summary of previous work is given with reference to important original papers.

Damped Electric Oscillations; solutions of differential equations for four types of oscillations: (1), *simple harmonic;* (2), *logarithmically damped;* (3), *linearly damped;* (4), *combined logarithmically and linearly damped.*—The mathematical treatment is based upon the analogy between the dynamical and electrical differential equations of motion. A simplification results by this method in the solution of the fourth case.

IN 1842 Joseph Henry discovered the oscillatory character of the discharge of a condenser. In 1853 Lord Kelvin[1] published the law of logarithmic damping for oscillatory currents. However, it was not until 1900 that rectilinear damping was discovered. The discharge of a condenser through a spark gap was investigated by F. Richarz[2] and W. Ziegler in 1900; they viewed the image of a spark gap in a revolving mirror; Fig. 1 is from their published drawing. In 1904, J. Zenneck photographed the spark with a Braun tube oscillograph and obtained

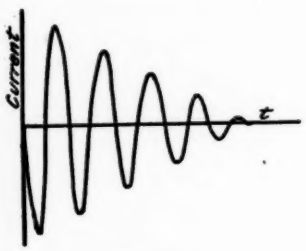

Fig. 1.

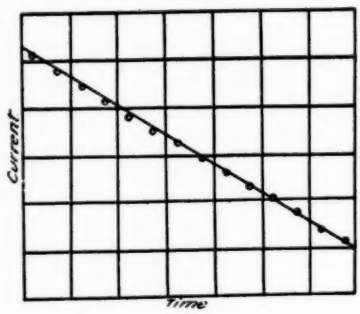

Fig. 2.

photographs of the turning points of the oscillations in the line of deflections. Fig. 2 shows the turning points plotted parallel to the time axis. In 1906 Heyweiller[4] attempted a mathematical solution for rectilinear damping.

[1] Phil. Mag., 1853, Ser. 4, Vol. 5, p. 393.
[2] Ann. d. Phys., 1, p. 468, 1900.
[3] Ann. d. Phys., 13, p. 822, 1904.
[4] Ann. d. Phys., 19, p. 646, 1906.

H. Barkhausen[1] published a solution in 1907, based upon the theory that the back E.M.F. across the spark gap, for the ideal case, is $e = a$ for the first half of an oscillation and $e = -a$ for the second half. The change in sign of the back E.M.F. at the end of each half cycle introduces a discontinuity, such that it becomes necessary to get a new solution for each half period; this is equivalent to an alternating change of origin. The E.M.F. circuital equation is stated as follows:

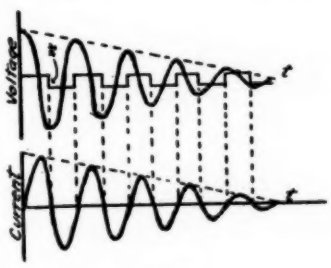

Fig. 3.

$$L\frac{di}{dt} + Ri = \frac{q}{C} \mp a,$$

where q is the charge on a condenser of capacity C, which discharges through an inductance L and resistance R (spark gap and circuit).

The equation reduces to

$$L\frac{d^2E}{dt^2} + R\frac{dE}{dt} = -\frac{1}{C}(E \pm a),$$

where

$$i = -\frac{dq}{dt} = -C\frac{dE}{dt}.$$

The two cases considered were: first, that where $R = R$; second, that where $R = 0$. The first case is the general one; the second case is that of simple harmonic motion with change of origin each half period.

The solution for the general case, given for the first half cycle is:

$$E - a = (E_0 - a)e^{-\frac{Rt}{2L}} \cos wt,$$

$$i = (E_0 - a)\sqrt{\frac{C}{L}}e^{-\frac{Rt}{2L}} \sin (wt + \theta),$$

$$W = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}.$$

The solution for the case where $R = 0$ is

$$E - a = (E_0 - a) \cos wt,$$

$$i = (E_0 - a)\sqrt{\frac{C}{L}} \sin wt,$$

$$W = \sqrt{\frac{1}{LC}}.$$

[1] Phys. Zeit., 8, p. 624, 1907.

In 1908 and 1911 Roschansky[1] published his work with the Braun tube. Photographs of the spark voltage characteristic are given with various metals as electrodes. Drawings showing the characteristics in the two extreme cases are given here. Fig. 4 shows the potential wave form

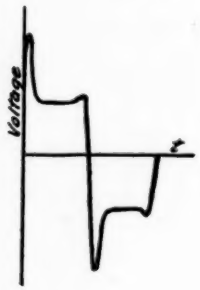

Fig. 4.

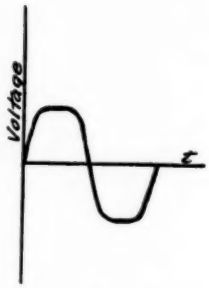

Fig. 5.

of the copper group, while Fig. 5 shows that of the magnesium group. The curves are said to hold for radio frequencies.

In 1911 E. L. Chaffie[2] published his work with the Braun tube, showing photographs of the current wave form at the spark. The photographs are remarkable and show the true linear decrescence of the current amplitude. Fig. 6 is a drawing of the spark gap damping curve which he gives. He says in regard to the form of the curve: "When the discharge is a pure arc, the separate oscillations are regular and practically sinusoidal, and the damping is linear. If the discharge is classed as a spark discharge, although the damping is linear, the separate oscillation loops are distorted as will appear presently."

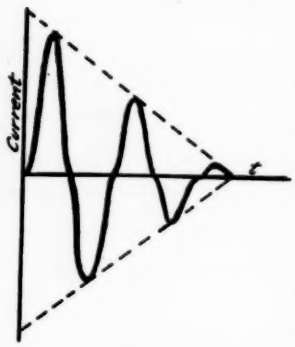

Fig. 6.

In 1915, A. Blondel[3] and Carbenay published their work with pendulums which were damped by constant bearing friction and by friction proportional to the velocity. They developed and applied a solution for pendulums from the same principles of harmonic motion which Barkhausen had used for electrical circuits. Their results show that the principle of origin change in harmonic motion is correctly applied to pendulums. (See Fig. 7.)

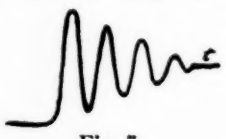

Fig. 7.

[1] Phys. Zeit., 9, p. 627, 1908.
Ann. d. Phys., 36, p. 281, 1911.

[2] Proceedings American Academy of Arts and Sciences, 47, p. 267, 1911–1912.

[3] La Lumiere Electrique, p. 194, Nov., 1915; p. 216, Dec., 1915; p. 241, Dec., 1915.

We know that slow motion pressure friction is fairly constant and that it always opposes motion; it must, therefore, be represented by the plus and minus sign. The equation expressing the oscillatory motion of a pendulum is given as:

$$A\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + C\theta = \mp K.$$

If only the pressure friction is present $B = 0$ and

$$A\frac{d^2\theta}{dt^2} = -C(\theta \pm K').$$

This equation represents S.H.M. with alternate change of origin. These investigators used a magnetic pendulum with a small mirror and a beam of light reflected on to a photographic plate. Constant friction was produced by the bearing pressure, and a strong magnetic field at right angles to motion gave damping proportional to the speed. In this way accurate tracings of the motion were made which show logarithmic and linear damping and combinations of both. Fig. 6 is taken from their published photographs. Application of this law is made to meters with pivot friction and air damping.

The next notable contribution to the subject was presented by John Stone[1] in 1914. This paper deals with the subject from a new viewpoint. Stone's method of attacking the problem is to build up an expression for the current which will give a linear decrescence of amplitude without bringing in a discontinuity. (See Fig. 8.)

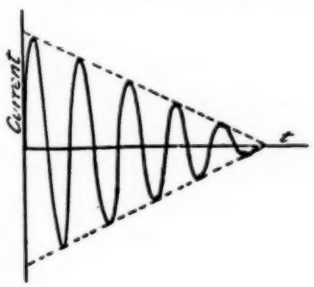

Fig. 8.

The expression for an amplitude of linear decrescence which he gives is $(A - Bt)$ and the current at any time t is given by $i = (A - Bt) \sin wt$. The assumptions are that A, B, and w are not functions of t, but constant quantities. The assumptions in regard to the spark resistance are that it is the instantaneous value of e/i and is made up of two components, one a dissipative resistance, the other a thermoelectric counter E.M.F.

The first paper deals with the case where the conductor resistance is zero, and all of the energy of the condenser charge is dissipated at the gap. The expression $i = (A - Bt) \sin wt$ for the current is applied to the circuital equation for the discharge of a condenser of capacity c through an inductance L and a spark gap of a so-called "resistance" r

[1] Proceedings I. R. E., 2, p. 307, 1914.

(undetermined). The circuital equation is written

$$L\frac{di}{dt} + ir = \frac{q}{C}, \qquad \text{where} \qquad i = -\frac{dq}{dt}.$$

The result follows:

$$r = \frac{2BL}{A - Bt}, \qquad w = \sqrt{\frac{1}{LC}}$$

and spark gap E.M.F. $= 2BL \sin wt$.

A complete discussion of the wave forms of the current and the gap E.M.F. is given in the paper.

In 1915[1] Stone published his second paper dealing with a solution for the general case. This paper is based upon the results of the previous one together with Kelvin's solution for logarithmic damping. The case is that of linear and logarithmic damping taking place at the same time. The conductor resistance R_1 is considered separate from the spark resistance r, in the circuital equation which follows:

$$L\frac{di}{dt} + R_1 i + ri = \frac{q}{C}, \qquad \text{where} \qquad i = -\frac{dq}{dt}.$$

If r (spark resistance) is zero the current is, from Kelvin's logarithmic theory,

$$i = I_1 e^{-R_1 t/2L} \sin wt,$$

$$w = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{2L}\right)^2}.$$

If both resistances are present in the circuit, Stone leaves the amplitude I undetermined and expresses the current in the same form as he did in the first paper, that is $i = I \sin wt$, where the value of I is to be determined by substitution in the above circuital equation. The value of r (spark resistance) is assumed to have the general value as determined in his former paper, that is inversely proportional to the current amplitude, or

$$r = \frac{d}{I} \text{ where } d \text{ is constant.}$$

By substitution of these conditions in the above circuital equation and by some rather complicated mathematical transformations Stone arrives at an expression for the current; it is given as follows:

$$i = Q_0 \frac{w_0^2}{w} e^{-a_1 t} \sin wt - \frac{2a_0 w}{a_1 \sqrt{a_1^2 + 4w^2}} \left\{ \sin\left(wt + \tan^{-1}\frac{a_1}{2w}\right) - \frac{\sqrt{a_1^2 w^2 + (a_1^2 + 2w^2)^2}}{w\sqrt{a_1^2 + 4w^2}} e^{-a_1 t} \sin\left(wt + \tan^{-1}\frac{a_1 w}{a_1^2 + 2w^2}\right) \right\},$$

[1] Proceedings I. R. E., 4, p. 463, 1916.

where

$$w_0 = \sqrt{\frac{1}{LC}}, \qquad w = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{2L}\right)^2}, \qquad a_1 = \frac{R_1}{2L} \qquad \text{and} \qquad a_0 = \frac{d}{2L}.$$

Oscillograms show that odd harmonics are present in the spark gap voltage, no doubt, due to the presence of ionization in the spark gap gases so that the current and voltage relation departs from Ohm's law. Quoting Stone: "To sum up, the proportionality of the instantaneous values of the gap voltage to the gap current, may notably depart from constancy, in any particular circuit, without sensibly modifying the frequency or mode of subsidance of the oscillations except perhaps near the end of a train of oscillations."

Outline of a Solution by Analogy.

The whole subject of "Damped Electric Oscillations" can be treated by the method of analogy. The parallelism between the dynamical and electrical equations for simple harmonic motion and logarithmically damped motion is well known. It is proposed to show: first, that a parallelism exists between the electrical and dynamical equations for linearly damped motion and for combined linearly and logarithmically damped motion; second, that the solutions in the last two cases named above are easily obtained, by analogy and that a greater simplification is obtained in the solution of the latter case than has hitherto been obtained.

The dynamical equations will be developed first and the electrical equations last, together with their solutions.

Oscillatory Motion and Dynamical Equations.

Oscillatory motion is most easily described in terms of the projection of a radius vector r, rotating with constant angular velocity w, about a center c. Let the radius vector make an angle θ with the X axis, passing through the center c such that $\theta = wt$, where t is the time from the instant when the radius vector coincides with the positive X axis. The motion may be described as taking place in either X or Y, thus: the displacement $x = r \cos wt$ and $y = r \sin wt$. (See Fig. 9.)

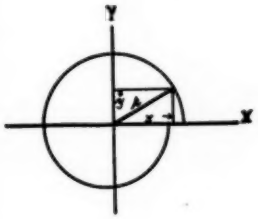

Fig. 9.

The point P at the end of r traces out some curve such as a circle, ellipse or some form of spiral. If there is no dissipation of energy during periodic motion, the auxiliary curves will be of the spiral family: the auxiliary curve for damping proportional to the speed being the logarithmic spiral and for constant damping, the rectilinear (Archimedes') spiral.

The differential equations expressing the acceleration in free periodic motion such as the cases above mentioned, are of the second order with coefficients which are either constants or expressible as functions of the time. If p and q are such coefficients, the general equation is written as follows:

$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = 0.$$

By reference to the auxiliary curves, the values of p and q can be determined for all classes of periodic motion, and their differential equations can be written. The types of motion which have been mentioned will be considered in the following paragraphs.

CIRCULAR HARMONIC MOTION (CASE 1).

In this case r is constant and the auxiliary curve is a circle. The well-known differential equation is

$$\frac{d^2y}{dt^2} + w^2y = 0. \tag{1}$$

LOGARITHMIC DAMPING (CASE 2).

In this case r decreases at a rate proportional to itself. The rate is expressed as follows:

$$\frac{dr}{dt} = -k_1r.$$

By integration

$$\log\left(\frac{r}{r_0}\right) = k_1t.$$

Solving for the radius vector, $r = r_0e^{-k_1t}$; therefore the auxiliary curve is a logarithmic spiral. The differential equation is

$$\frac{d^2y}{dt^2} + 2k_1\frac{dy}{dt} + (k_1^2 + w^2)y = 0. \tag{2}$$

RECTILINEAR DAMPING (CASE 3).

The radius vector r decreases at a constant rate $-k$.

$$\frac{dr}{dt} = -k.$$

By integration $r - r_0 = -kt$. The radius vector $r = r_0 - kt$, therefore, the auxiliary curve is a rectilinear spiral.

Writing $y = (r_0 - kt)\sin wt$, the differential equation is found to be

$$\frac{d^2y}{dt^2} + \frac{2k}{r_0 - kt}\frac{dy}{dt} + \left[\frac{2k^2}{(r_0 - kt)^2} + w^2\right]y = 0. \tag{3}$$

LOGARITHMIC AND RECTILINEAR DAMPING COMBINED (CASE 4).

The radius vector is proportional to the radius vector of each case separately and is proportional, therefore, to their product when the cases are combined; it is equal, therefore, to a constant times their product. The constant may be taken as unity in our case. The radius vector is

$$r = (r_0 - kt)e^{-k_1 t}.$$

Writing $y = (r_0 - kt)e^{-k_1 t} \sin wt$, the differential equation is found to be

$$(4) \quad \frac{d^2y}{dt^2} + 2\left[k_1 + \frac{k}{r_0 - k_1 t}\right]\frac{dy}{dt} + \left[2\frac{k^2}{(r_0 - kt)^2} + 2k_1\left(\frac{k}{r_0 - kt}\right) + k_1^2 + w^2\right]y = 0.$$

ANALOGOUS ELECTRICAL EQUATIONS.

When the current follows the simple harmonic law for the discharge of a condenser of capacity C through an inductance L, the differential equation is

$$(5) \quad \frac{d^2i}{dt^2} + \left(\frac{1}{LC}\right)i = 0.$$

The term $1/LC$ corresponds to W^2 in the parallel dynamical equation (1).

Likewise, when the current follows the law of logarithmic damping, due to a resistance R in the discharge circuit, the differential equation is

$$(6) \quad \frac{d^2i}{dt^2} + 2\left(\frac{R}{2L}\right)\frac{di}{dt} + \left(\frac{1}{LC}\right)i = 0.$$

The term $R/2L$ corresponds to K_1 and the term $1/LC$ corresponds to $k_1^2 + w^2$ in the parallel dynamical equation (2).

Similarly, the differential equation for the current, when it follows the law of linear damping, can be shown to be analogous to the dynamical equation (3), provided, that the usual manner of expressing the E.M.F., circuital equation

$$L\frac{di}{dt} + ir = \frac{q}{C}$$

holds in this case. Here, q is the quantity of electricity in the condenser of capacity C discharged through an inductance L and a spark gap, r being an undetermined function such that ir expresses the back E.M.F. of the spark gap.

It is only necessary to differentiate the above equation to get the

analogous electrical equation. The equation is

$$\frac{d^2i}{dt^2}+\frac{r}{L}\frac{di}{dt}+\left[\frac{1}{L}\frac{dr}{dt}+\frac{1}{LC}\right]i=0.$$

Thus by analogy, one can choose r such a function of the time as to make the solution of this equation correspond to linear damping. For, let r/L correspond to $2k/(r_0-kt)$ in (3); its value will be $2k/(I_0-kt)$ in this case, where I_0 is the initial current amplitude and K is the rate at which the current amplitude decreases; also letting $1/LC$ correspond to W^2, the analogy is completed because

$$\frac{1}{L}\frac{dr}{dt}=\frac{2k^2}{(I_0=kt)^2},$$

which corresponds to $2k^2/(r_0-kt)^2$ in (3).

The complete analogous electrical equation, therefore, writing

$$\frac{r}{2L}=\frac{k}{I_0-kt}$$

and

$$2\frac{k^2}{(I_0-kt)^2}=2\left(\frac{r}{2L}\right)^2$$

is

$$\frac{d^2i}{dt^2}+2\left(\frac{r}{2L}\right)\frac{di}{dt}+\left[2\left(\frac{r}{2L}\right)^2+\frac{1}{LC}\right]i=0. \tag{7}$$

The solution of this equation is

$$i=(I_0-kt)\sin\left(\sqrt{\frac{1}{LC}}\right)t.$$

When the current undergoes both linear and logarithmic damping simultaneously, its differential equation can be shown in the same manner to be analogous to the dynamical equation (4), provided that the E.M.F. circuital equation be written for the mutual case as follows:

$$L\frac{di}{dt}+Ri+ri+E=\frac{q}{C},$$

where r has the same value as determined in the preceding case and E, a mutual term, is an undetermined function. Thus, by differentiation the resulting equation becomes analogous to the dynamical equation (4), if one writes

$$\frac{dE}{dt}=Zi.$$

The equation thus derived is

$$\frac{d^2i}{dt^2}+\left(\frac{R}{L}+\frac{r}{L}\right)\frac{di}{dt}+\left[\frac{1}{L}\frac{dr}{dt}+\frac{Z}{L}+\frac{1}{LC}\right]i=0.$$

By analogy one can now choose Z/L such a function of the time as to make the solution of the equation correspond to combined logarithmic and linear damping. Thus, let

$$\frac{Z}{L} = 2\left(\frac{R}{2L}\right)\frac{k}{I_0 - kt}$$

to correspond to $2k_1(k/(r_0 - kt))$ in (4). Writing

$$\frac{r}{2L} = \frac{k}{I_0 - kt}$$

as in the previous case,

$$2\left(\frac{R}{2L}\right)\frac{k}{I_0 - kt} = 2\left(\frac{R}{2L}\right)\left(\frac{r}{2L}\right)$$

and the analogous electrical equation is

$$(8)\quad \frac{d^2i}{dt^2} + 2\left(\frac{R}{2L} + \frac{r}{2L}\right)\frac{di}{dt} + \left[2\left(\frac{r}{2L}\right)^2 + 2\left(\frac{R}{2L}\right)\left(\frac{r}{2L}\right) + \left(\frac{1}{LC}\right)\right]i = 0.$$

The solution of this equation, therefore, is

$$i = (I_0 - kt)e^{-\frac{Rt}{2L}}\sin\left(\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\right)t,$$

which is simpler in form than any hitherto derived for the combination case. Equation (8) reduces to any one of the three previous cases as follows:

first, when R and r are zero it reduces to (5), namely, simple harmonic oscillations;

second, when r is zero it reduces to (6), namely, logarithmically damped oscillations;

third, when R is zero it reduces to (7), namely, linearly damped oscillations;

fourth, as it stands it is a combination of the three cases.

It is of importance to determine the resulting E.M.F. expression for the spark. This is seen to be expressed as $ir + E$ from the E.M.F. equation already considered. It is now necessary to determine E. The rate of change of E has already been found, by analogy, to be $dE/dt = Zi$, where Z was found to be

$$Z = 2LK\frac{k}{I_0 - kt}.$$

The value of E can be found by integration as follows:

$$E = \int Zidt = 2k_1kL\int e^{-k_1t}\sin wtdt,$$

$$E = -\frac{2k_1kL}{w^2 + k_1^2}e^{-k_1t}(k_1\sin wt + w\cos wt).$$

The total E.M.F. across the gap, expressed as $ri + E$, is now determined in terms of the circular functions and is,

$$ir + E = 2kL\frac{w}{\sqrt{w^2 + k_1^2}}e^{-k_1t}\sin\left(wt - \tan^{-1}\frac{k_1}{w}\right).$$

This expression gives the gap E.M.F. as a sinusoid with a logarithmically decaying amplitude. Moderately large amplitudes in the odd harmonies of the gap E.M.F. need not necessarily affect the current curve to an appreciable extent, so that, from a practical standpoint these results, based upon the above assumptions, do have considerable value so far as the current relations are concerned.

This, however, does not exactly fit the facts as the analyses of both current and gap E.M.F. wave forms which have been made to determine the harmonics present, show. These show that the odd harmonics are present in both current and gap E.M.F., especially in the latter, and produce distortions in the wave forms. Before a complete solution is possible for the circuits, parts of which are made up of ionized gas, one must use some function expressing the departure from Ohm's Law for such circuits.

WASHINGTON STATE COLLEGE,
December, 1920.

SOME PECULIARITIES OF POLARIZATION AND ENERGY DISTRIBUTION BY SPECULUM GRATINGS.

By L. R. Ingersoll.

Synopsis.

Intensity Minima in Spectra of Speculum Gratings.—A series of intensity minima, resembling absorption lines, are to be found in the continuous spectrum of a speculum grating at such wave-lengths as are being tangentially diffracted in other orders. They are limited to the azimuth of polarization with electric vector perpendicular to the rulings and appear best in such gratings as show strong spectra in several orders. While they appear in the visible spectrum for a deeply ruled grating, in general they may be observed only for longer wave-lengths—say between .8 μ and 2 μ. Although simple energy considerations furnish a fairly satisfactory explanation of the phenomena, it is highly desirable that the Rayleigh-Voigt grating theory be modified and extended to fit these and similar facts or else that a new theory be developed.

Practical Importance to the Spectroscopist.—It is highly desirable that work on the Zeeman effect or other polarization phenomena with speculum gratings be arranged—as may be easily done—so as to avoid this effect. The same may be said of infra-red absorption measurements.

IT is a well-known fact that grating spectra show certain peculiarities not exhibited by prismatic spectra, such as ghosts, abnormalities of polarization and some others. To these there may now be added—for speculum gratings—another peculiarity in the form of a series of marked minima of intensity in the continuous spectrum, superficially resembling absorption lines. These are confined to the azimuth of polarization in which the vibration (electric vector) is perpendicular to the rulings, and are as a rule much more prominent in the early infra-red region—wave-length from .8 μ to 2 μ—than in the visible spectrum. This probably accounts for their not having been generally observed before.[1] Another reason is that the effect is not likely to show very well in a grating of the sort which would ordinarily be chosen for spectroscopic work, *i.e.*, one having, among other properties, that of concentrating its energy in one spectrum, while it is marked in gratings which give bright second and higher orders as well as the first.

The phenomenon was not wholly unlooked for when the problem was

[1] The dark bands observed by Wood (Phil. Mag. (6), **4**, 396, 1902, and **23**, 310, 1912) for an abnormal speculum grating and particularly for gratings ruled on silver, while unquestionably closely related to these minima cannot, from the description, be certainly identified with them.

taken up, but was not just what was expected. In a previous paper[1] the writer has described experiments which show that the directly reflected light or central image from a speculum grating is markedly deficient in light of such wave-length as is, in the first or higher order spectrum, emerging tangentially from the grating. The effect is confined to the azimuth of polarization already mentioned, viz., with electric vector perpendicular to the rulings, and is in general accord with the Rayleigh-Voigt grating theory.[2] This theory goes on to predict that, for this same azimuth of polarization one should find in the first order spectrum, say, at such wave-lengths as are being tangentially diffracted in the second or higher order spectra, sharp *maxima* of energy. Wood[3] has found such an effect in the case of a special grating ruled on silver. The phenomena noted here are, with one exception, just the reverse of this, viz., *minima* instead of maxima.

A rather simple spectro-bolometric apparatus served very well in the measurement of this effect. Light from a special tungsten strip filament lamp[4] was focused by a concave mirror (of approximately 30 cm. focal length and 8 cm. aperture) on a slit at the principal focus of another mirror, from which the light then fell in a parallel beam on the grating. The diffracted light passed through a large (5 cm. aperture) Wollaston double image prism of calcite, which separated the beam into the two azimuths of polarization, perpendicular and parallel to the rulings, respectively. These spectra were focused by another mirror on the two strips of a special bolometer so that in this way the energy in each state of polarization could be investigated independently. A sensitive 8-coil Thomson galvanometer at a distance of approximately 3 meters served in connection with the bolometer to measure the energy at any given wave-length, which energy is expressed, in this paper, in terms of the galvanometer deflections. To avoid the effect of overlapping spectra a 25° glass prism was placed between the lamp and first concave mirror. The latter was adjustable so that the slit could be illuminated by a block of radiation, perhaps .2 μ wide in wave-length, taken from any part of the resulting (impure) spectrum.

The illuminating system of lamp, prism, two mirrors, and slit, was rigidly mounted and arranged so that it could be rotated with the grating about the axis of the spectrometer. In studying the energy distribution the incidence was fixed at any desired angle, and the whole system then

[1] Astrophys. Jour., 51, 129, 1920.

[2] Proc. Roy. Soc. (A), 79, 399, 1907. Gött. Nachr. Math. Phys. Kl. (1911), p. 40.

[3] Phil. Mag. (6), 23, 310, 1912.

[4] These lamps are now obtainable from the Nela Laboratory. They are invaluable in spectroscopic work where an intense constant source of this nature is desired.

rotated so that the angle of diffraction of the radiation falling on the bolometer could have, in turn, any value up to 90°.

Gratings.—Through the kindness of Professor Michelson and Professor Gale four gratings were specially ruled by Mr. Fred Pearson of Ryerson Laboratory and furnished for this investigation. They all had ruled surfaces approximately five centimeters square, and were of the following spacings: No. 1, 8709 lines to the inch—$S = 2.916\,\mu$; No. 2, 7620 lines per inch—$S = 3.333\,\mu$; No. 3, 10160 lines per inch—$S = 2.500\,\mu$; No. 4, same spacing but ruled with at least ten times the pressure, giving a comparatively very deep cut. No. 5 was a Hopkins grating of 15,000 lines per inch ($S = 1.693\,\mu$), with a quite extraordinary concentration of energy in one spectrum.

The results are shown graphically in Figs. 1 to 4. These are in reality energy curves plotted with angle of diffraction as abscissa instead of

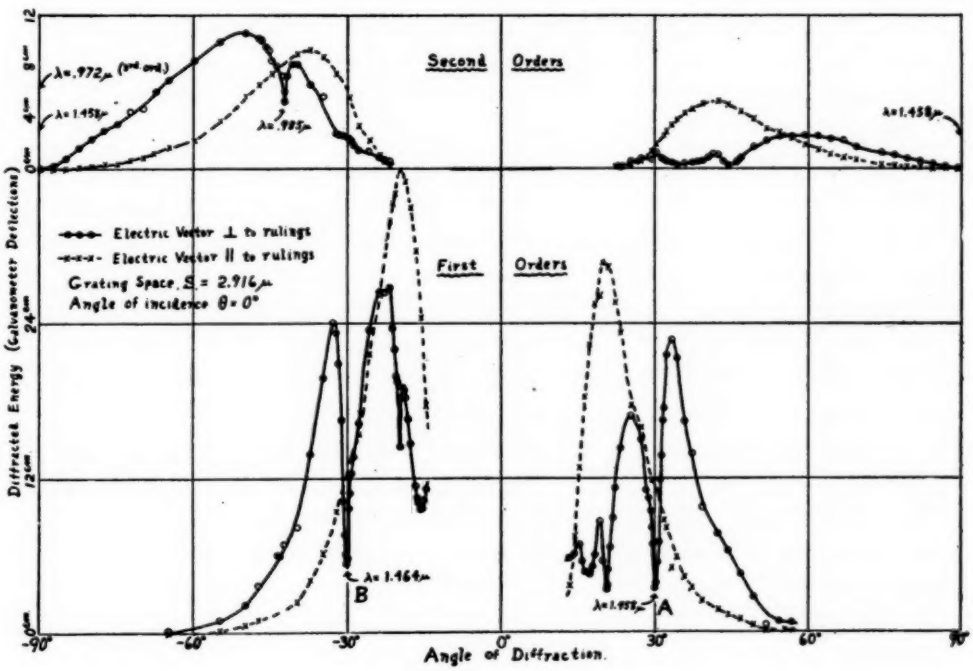

Fig. 1.

Energy curves (of a tungsten filament) for both azimuths of polarization, as determined with a speculum grating (No. 1) of 8,709 lines per inch. Light incident normally.

wave-length, as in the ordinary case. No correction of any sort has been applied. The plus and minus signs in connection with the angle of incidence or diffraction are used to indicate the two sides of the grating normal.

The minima at or very near angle 30° in Fig. 1 are very marked. Now since sin 30° = 1/2, the same wave-length occurs here in the first order as is emerging tangentially in the second: how closely these wave-lengths check is indicated. There are also secondary minima in the neighborhood of 20° and 15° for wave-lengths which are tangential in the third and fourth orders respectively, though the agreement is not so good in this case.

Referring now to the upper part of the figure, showing the second order, it will be remarked that there is a considerable amount of energy in this order and it is particularly to be noted that as the angle of emergence approaches 90° the proportion of radiation of perpendicular polarization to parallel becomes larger and larger. This is in accordance with the Rayleigh theory and is a fact which can be readily observed with almost any speculum grating by examining the tangentially emergent light with a nicol prism.

A simple and fairly satisfactory explanation of these minima may be based on this last mentioned fact, viz., the relatively large amount of perpendicularly polarized radiation coming obliquely from the grating (although to account for the excess of energy of this state of polarization we must, for the present, rely entirely on the Rayleigh theory; no purely physical reason can, as yet, be adduced). For this excess must be made up by a deficiency at this same wave-length region in other spectra and since the dispersion at nearly tangential emergence is very great—the change in the sine of the angle between 80° and 90° is only one eighth that between 40° and 50°—the obliquely diffracted radiation comes from a very limited spectral region. This accounts for the (usually) narrow minima found at this wave-length in other orders.[1]

A rather critical test of this reasoning was applied by tilting the grating slightly so that the incidence was about 2 1/2° instead of zero. The wave-length tangential in the second order on one side would then be somewhat shorter and on the other, somewhat longer than before. The minimum in the first order should then appear split into two, neither as pronounced as the original one, and the two might furthermore be expected to be of different intensities, corresponding to the difference between the two second order spectra. These predictions are all seen to be amply verified in Fig. 2.

[1] It might be thought that this point could be made more evident by plotting energy in terms of wave-length rather than angle of diffraction, in which case there should appear a sharp maximum at tangential emergence which would correspond exactly to the minimum in some other order at the same wave-length. This does not show satisfactorily, however, because there is so much loss of energy at or near tangential emergence. That is to say, at 70° light may come from all parts of a furrow, whereas at 89° the shielding of one furrow by another is such that the only light emerging comes from the tips.

At other angles of incidence and for other gratings the minima may have markedly different shapes, as Fig. 3 shows. A casual study of these curves will show the correspondence between the wave-lengths of the minima and the radiation tangentially emergent in some other order. While in most cases this order is higher than the one under investigation,

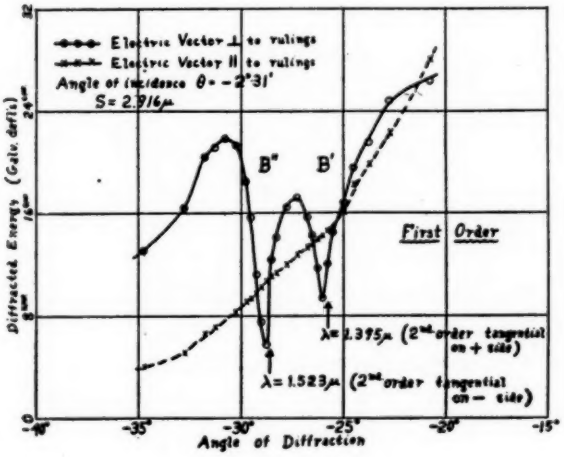

Fig. 2.

Detailed study of minimum *B* (Fig. 1) for an incidence of −2°31′.

a number of instances have been found in which the minimum in the first order on one side corresponds to tangential emergence in the first order on the other. It will be noted that a characteristic of all the minima is a sharp and rapid rise of intensity on the side of longer wave-lengths (*i.e.*, when this wave-length in the second order, say, has passed entirely from the grating) while the rise on the short wave-length side may be very much delayed. In some cases indeed it does not occur at all and the minimum degenerates into a sharp change in intensity.

A rather remarkable case of this is shown in Fig. 4. At first sight it would be thought that this is conclusive evidence for the Rayleigh theory, but a little closer investigation causes one to doubt this and indeed to consider it as merely a special case of the preceding phenomena. It will be noted that the detailed study of this small maximum, for the three angles of incidence, shows that the *sharp side*, and not the peak of the minimum, corresponds exactly to the above theory. Why the energy should fall off rapidly on the right-hand side as it does, giving to the whole the appearance of a fairly sharp maximum, cannot be explained

save by remarking that it is undoubtedly due to the groove form, which doubtless accounts in turn for the extraordinary concentrating power of this grating.[1]

Visual Observations.—As already remarked, most gratings show this effect very much better in the infra-red than in the visible spectrum. An exception, however, is grating No. 4 which has very deep furrows and which gives visible dark bands in excellent agreement with the above

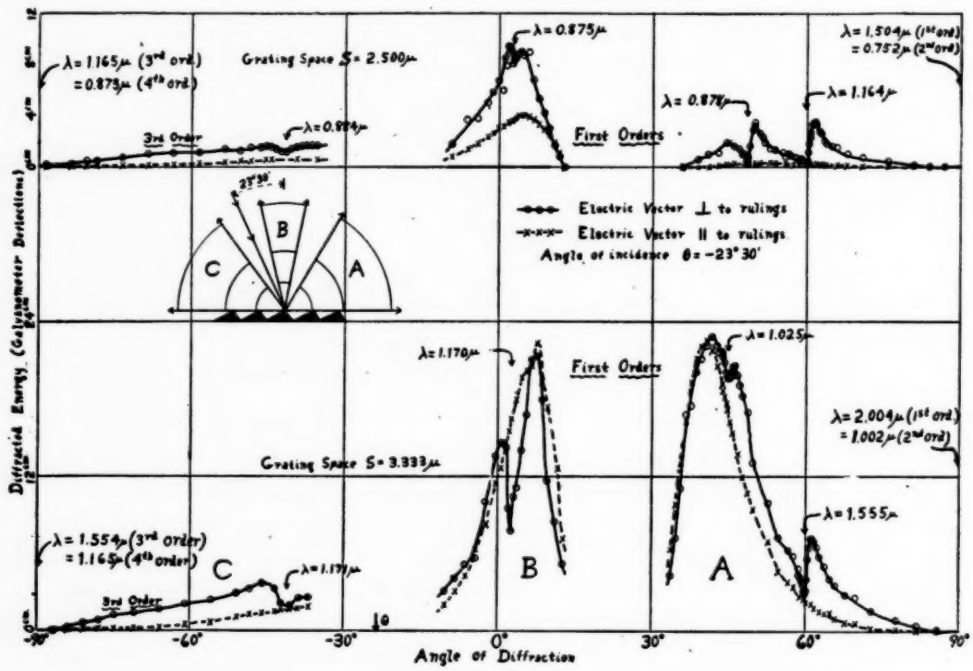

Fig. 3.

Energy curves (of tungsten filament) for both azimuths of polarization, as determined with speculum gratings (Nos. 2 and 3) of 7,620 and 10,160 lines per inch. Angle of incidence, −23°30′.

reasoning. As a matter of fact the agreement is very much better than in the case of the infra-red minima for this same grating. The dark bands observed by Wood, as already mentioned, are doubtless also manifestations of this same general phenomenon and may indeed be identical with the minima found here, although his data does not afford a quantitative check in this case.

[1] It is perhaps a fair question to ask if the phenomena observed by Wood (Phil. Mag. (6), 23, 315, 1912) were not identical with this, a fact which, of course, could only be cleared up by careful investigation, if possible, as to whether the *center* or *edge* of the sharp maximum he observed checks with the theory.

Discrepancies.—While the foregoing explanation of the phenomena as described seems fairly satisfactory, at least to the writer, there are certain other effects observed in this connection which must at present remain totally inexplicable. It will be noted in the curves that in some

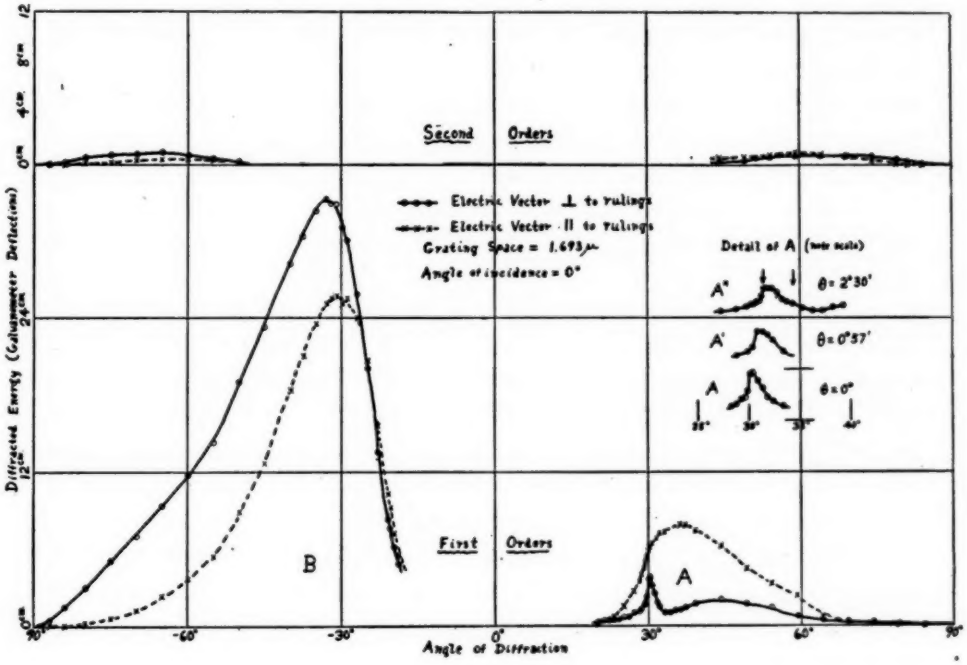

Fig. 4.

Energy curves for a grating (No. 5) of 15,000 lines per inch. Light incident normally.

cases the wave-length at the point of the minimum is slightly longer than that tangentially emergent in the other order. While in the figures shown the discrepancy does not exceed 2 per cent., there are cases in which this is even more.

Considerable study and discussion[1] of this point has failed to yield any very satisfactory explanation, but the outstanding phenomena will be briefly described in order to aid anyone in fitting an explanation to the facts. They are: (*a*) The discrepancies are, when they exist, always in the same sense, *e.g.*, the wave-length at the minimum is always *longer*

[1] Professor Sparrow suggests in explanation of this, an interaction between the light vibrations and the conduction currents they occasion in the furrowed metal on reflection. Professor Wood believes that this is another point of evidence to be added to his results (Phil. Mag., July, 1919) on reflection from films of granular or furrowed surfaces which seem to demand for explanation some new principle in optics. It is probable that the best method to clear this up is to use either very long infra-red wave-lengths or short electromagnetic waves with gratings which can be ruled on a correspondingly large scale.

than that tangentially emerging in the other order. If it were shorter it could be easily explained. (*b*) The disagreement is greater, as a rule, when the light is incident normally or nearly so. In some cases a grating which shows a discrepancy of 2 or 3 per cent., at normal incidence will give results checking to a small fraction of a per cent., when the incidence is increased to 30° or so. (*c*) The discrepancy is most prominent in deeply furrowed gratings. (*d*) In the one grating No. 4 which shows both infra-red and visual minima the discrepancy is as already remarked, much greater in the infra-red than in visual measurements,—the agreement indeed, in the latter case, frequently reaching one part in a thousand.

It may be remarked on this latter point that the phenomena in the infra-red region under investigation doubtless depends on the furrow as a whole, whereas in the visual results probably only corresponding facets of the furrows, or possibly the thin edges, come into account. Wood found that the slightest rubbing of the gratings ruled on silver fundamentally altered the polarization effects he observed visually, while the writer in investigating for longer wave-lengths the phenomena for the central image found that they were changed in no way, even when the grating was polished with Vienna lime or rouge. We may accordingly then believe that when the furrow exceeds a certain depth or degree of roughness, as in grating No. 4, it can no longer act as a whole even on the longer wave-lengths and hence the results in this region are somewhat confused. The visual results, however, if concerned with only some small but similar part of each furrow may be quite distinct and regular and this seems to be the case. It may be added that for this deeply cut grating some trace of these polarization abnormalities are observable in the other azimuth of polarization.

Conclusion.—The minima of intensity for polarization (electric vector) perpendicular to the rulings which occur either in the central image or in any spectrum at such wave-lengths as are tangentially emergent in some other spectrum are probably characteristic of all speculum gratings. They are, however, prominent only in gratings cut with a poor diamond point so that considerable light is diffracted in all of the orders. While simple conservation-of-energy reasoning will account for the effects in a general way, a new grating theory, or substantial modification of the Rayleigh-Voigt theory, is undoubtedly needed to explain comprehensively these and similar phenomena. From a practical standpoint it is eminently desirable in working on polarization phenomena, *e.g.*, the Zeeman effect, with a speculum grating, to avoid the region of such wave-lengths as might show the effect described here. In studying infra-red absorption spectra it would also be well to have this point in mind, for while the

minima would be only half as prominent with unpolarized light they might still give rise to false absorption observations. A slight change in the angle of the grating would, of course, at once show such measurements as spurious.

In conclusion the writer wishes to express his indebtedness to the Rumford Fund for assistance in conducting this investigation.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN,
January, 5 1921.

CONDUCTIVITY OF INSULATING MATERIALS NEAR THE BREAKDOWN VOLTAGE.[1]

BY J. E. SHRADER

SYNOPSIS

Conductivity of Insulating Materials up to Breakdown Voltages.—(*a*) In the *method of measurement* used, the recording instruments were protected from injury in case of puncture, by placing an insulated hot cathode rectifier in series with the sample being tested. This enabled the current through the sample to be limited and also controlled by controlling the temperature of the cathode of the rectifier. When with a given applied voltage the current did not reach a constant maximum value upon raising the temperature of the cathode, the voltage had reached the breakdown point. (*b*) *These classes of insulating materials* were found whose conductivity, respectively, (1) obeys Ohm's law throughout; (2) obeys Ohm's law up to near breakdown and then increases at an accelerated rate; and (3) increases over the whole range of voltage at an accelerated rate to breakdown. In general, material in class 3 have the greatest dielectric strength; these in class 1, the least. Results are given only for these materials, untreated cement paper, paraffined fish paper, and black treated cloth, which were selected as representative, respectively, of the three classes.

MOST of the data on the conductivity of insulating materials have been obtained from observations of tests made at comparatively low potential stresses. It is quite well known that the behavior of all insulating materials under varying voltages is not the same. Some obey Ohm's Law while others increase in conductivity with voltage at a greater rate than in accordance with Ohm's Law.

Any data to be of service should be taken under the voltage conditions to which the materials are subjected. If material is to be used not merely for its resistance to the passage of a current but to stand high electrostatic stress and prevent breakdown then this material should be studied under stresses up to and in the neighborhood of actual breakdown.

To test insulating materials over a range of voltage up to the breakdown without danger to the current measuring instruments the following scheme shown by the diagram, Fig. 1, was devised. The D.C. voltage of 12,000 volts is obtained from the rectification of the A.C. voltage from the secondaries of two transformers AB and BC, by the hot cathode rectifier E. The primaries of the transformers are connected in parallel to an auto transformer by which the voltage is varied. The secondaries

[1] Paper read before the Physical Society at Cleveland, Nov. 25–27, 1920.

are connected in series and the terminals A and C are connected to the anodes of the two-anode rectifier E. The middle point B is connected through a second rectifier F, and the sample H to the cathode of the rectifier. The cathode of this rectifier is heated by the stepdown transformer I connected to a 110-A.C. source. A capacity G consisting of ten One-M.F. condensers connected in series is connected across the

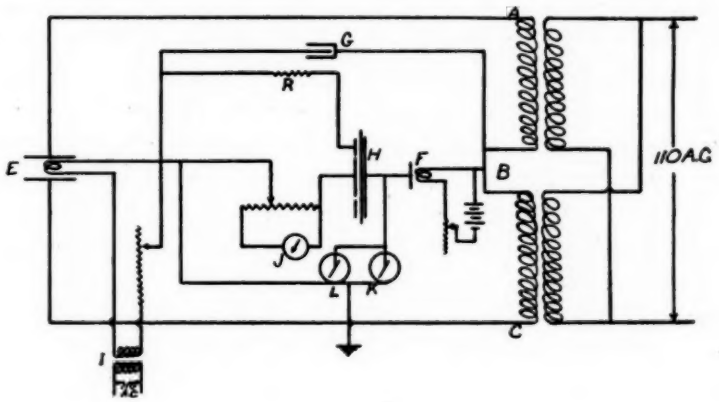

Fig. 1.

sample to maintain a constant voltage. The current through the sample is read by the galvanometer J which is provided with an Ayrton Shunt. The voltage across the sample is read by two electrostatic voltmeters K and L, one reading to 3,500 and the other to 15,000 volts.

The new feature in this arrangement is the insertion of the hot cathode rectifier F between the high tension electrode of the sample and the middle point B of the transformers. This rectifier with its battery and regulating resistance is well insulated. This rectifier or valve tube as it is used in this case serves as a protective device for the galvanometer. With the transformers excited no current can pass through the sample until the cathode of the rectifier F is heated. The value of the current which can pass through the tube is dependent upon the temperature of the filament, which is controlled by the variable resistance. Then for any given testing voltage the temperature of the cathode is gradually increased, observing at the same time the current indicated by the galvanometer. When there is no further increase of the current upon further increasing the temperature of the cathode of the rectifier it is known that the current is limited by the insulation and not by the valve. By this method one has complete control at all times of the value of the current which may flow through the instruments. If the current does not reach a steady value upon opening the valve it is a good indi-

cation that the sample is near the point of breakdown. Even in the case of actual breakdown, with the valve in the circuit, the current is limited to such a value that damage to the instruments is prevented.

For a study of the conductivity of insulating materials up to the point of breakdown three samples from the same sheet of material were prepared. As might be expected by one familiar with insulating materials, there is considerable variation in conductivity among these samples and this is shown by the curves which follow. The tests were made with a guard ring to prevent surface leakage to the measuring instruments. Close contact of the electrodes with the sample was obtained by the use of powdered graphite. The lower electrode was a round shallow tray filled with powdered graphite and smoothed off with a straight edge to a depth of about two millimeters. Upon this was placed the sample to be tested. Two concentric cylindrical glass rings were fastened with shellac upon the upper surface of the sheet material. The space between the rings occupied by the guard ring and the surface inside the inner ring were covered with the powdered graphite upon which were firmly pressed the guard ring and the central electrode respectively. The resistance of the graphite layer itself was negligible being only a few ohms. The contact, especially for an unpolished surface, was better than a contact of mercury since the powder could be rubbed into the small surface irregularities. That this was true was actually proven by trial.

For the materials tested the current came to practically a steady value in two to five minutes after the application of voltage. Hence the values used in this work were obtained after a steady reading of the galvanometer was observed.

Tests were made on a variety of commercial insulating materials, observations being made from comparatively low voltage to breakdown voltages for the particular sample. From the results obtained all these materials may be divided into three classes:

1. Materials whose conductivity varies directly with voltage according to Ohm's Law.

2. Materials whose conductivity varies according to Ohm's Law up to the nieghborhood of breakdown after which the conductivity increases at an accelerated rate to breakdown.

3. Materials whose conductivity over the whole range increases at an accelerated rate to breakdown.

Of quite a number of materials tested three have been chosen which illustrate the three classes. Fig. 2, untreated cement paper, is representative of the first class. This shows a straight line relationship between conductivity and voltage up to breakdown. Fig. 3, paraffined

fish paper, illustrates the second class. Here the straight line relationship holds up to the neighborhood of breakdown where there is a marked increase in the slope of the curve after which breakdown soon occurs. Fig. 4, for black treated cloth represents the more numerous materials

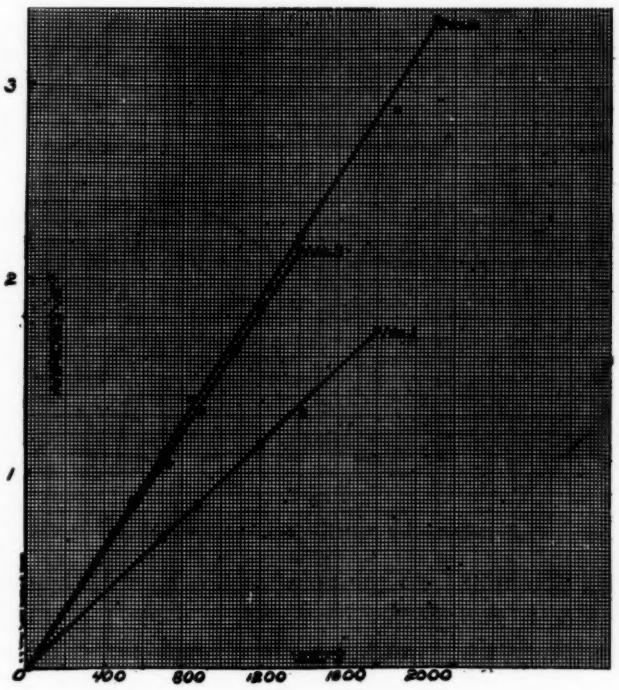

Fig. 2.

Untreated cement paper. Thickness =.015 in.

of the third class. This curve shows that there is an increasing slope to the curve from low voltages up to the point of breakdown.

From a comparison of calculated specific resistances of the various materials tested there is no apparent relation between specific resistance and breakdown voltage. It is quite apparent that the curves for Class 1, give no indication of breakdown. In Class 2, the breakdown point is clearly indicated by the upward slope just before breakdown. In Class 3, we might say that breakdown is liable to occur after the slope of the tangent to the curve exceeds an arbitrary angle determined from experience. From a comparison of the voltage gradient at breakdown, the results show that the materials in class 3 have the greater dielectric strength and those of Class 1, the poorest. Class 1, has the characteristics of a conductor in that the resistance is independent of voltage while Class 3, has no true resistance but only apparent resistance which

is a function of the applied voltage. In class 1 the current is no doubt due to the free electrons in the material under the directive force of the applied potential. In Class 3, the number of free electrons is a function

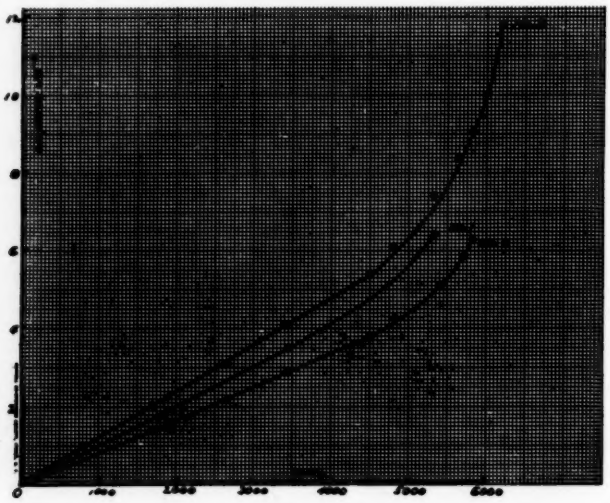

Fig. 3.

Paraffined fish paper. Thickness =.015 in.

of the applied potential. At the point of breakdown, then the number of free electrons is enormously increased, in Class 1 breakdown takes place very suddenly, while in Class 3, the change is more gradual.

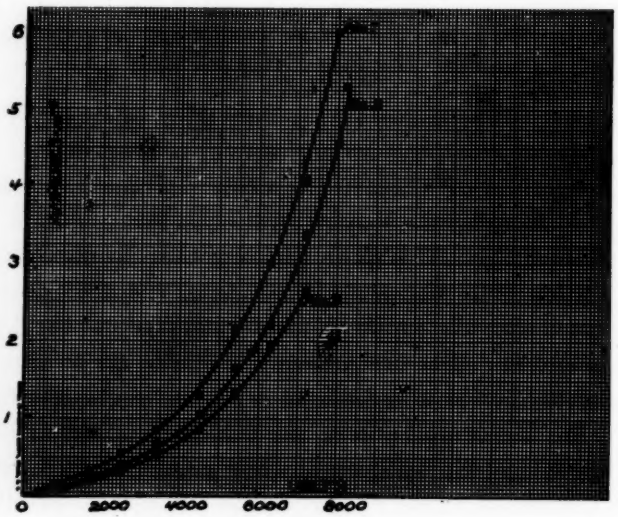

Fig. 4.

Treated cloth (black) No. 1010. Thickness =.011 in.

While no claims are made that the work here reported is complete, considerable useful information has been obtained as to how insulating material behaves at potentials near the breakdown. The method devised may be useful also for further work in investigating the nature of breakdown of insulating materials.

RESEARCH LABORATORY,
WESTINGHOUSE ELECTRIC AND MANUFACTURING CO.,
EAST PITTSBURGH, PA.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE STANFORD UNIVERSITY MEETING, JANUARY 22, 1921.

THE 107th meeting of the American Physical Society was held in the lecture room of the Department of Physics, Stanford University, January 22, 1921, at two o'clock. Professor Fernando Sanford presided. About fifty persons were in attendance.

The following program was presented:

The Balmer Series of Hydrogen and the Quantum Theory of Line Spectra. R. T. BIRGE.

Charts as an Aid in Teaching Electricity. F. J. ROGERS.

Damped Electrical Oscillations. R. B. ABBOTT.

A Thermoelectric Motor. PAUL KIRKPATRICK.

A Wave Composition Model. JOSEPH G. BROWN.

Diurnal Variations in Terrestrial Magnetism and in the Electrical Potential of the Earth. FERNANDO SANFORD.

Sunspots and Terrestrial Magnetic Storms. FERNANDO SANFORD.

Note on the Fahrenheit Scale. FLORIAN CAJORI.

Nature of Ionization in the Point to Plate Discharge. EVALD ANDERSON.

An Explanation of X-Ray Diffraction Patterns from Rolled Metals. C. T. DOZIER.

The Concentration of Monochromatic X-Rays by Crystal Reflection. ELMER DERSHEM and C. T. DOZIER.

At the conclusion of this program, Professor D. L. Webster showed and explained his high potential X-ray outfit.

Thirty-six persons were present at the dinner served at Stanford Union at six-thirty o'clock.

E. P. LEWIS,
Local Secretary for the Pacific Coast.

THE BALMER SERIES OF HYDROGEN, AND THE QUANTUM THEORY OF LINE SPECTRA.

BY RAYMOND T. BIRGE.

THE purpose of this paper is to make a rigid comparison between the experimental results for the Balmer series of hydrogen, and the theoretical developments of the quantum theory of line spectra. In particular,

Merton's experimental results for the separation, half-width, and relative intensity of the H_α and H_β doublets are examined in the light of the recent Bohr-Kramers' work on the intensity of the fine structure components, as estimated on the basis of the "selective" principle.

It is concluded that, by assuming a general electric field of 100 volts per cm., all of Merton's experimental results check quantitatively with theory. The frequencies of all the lines of the Balmer series are then fitted into the Bohr relativity formula, after the fine structure has been analyzed and the "circle" line computed, using the Sommerfeld theory to determine the position of the fine structure components, and the Bohr-Kramers' theory to determine the relative intensities. The Rydberg constant for hydrogen is calculated to be 109,677.7 $\pm$ 0.2, and the constant for a nucleus of infinite mass 109,736.9 $\pm$ 0.2.

The paper concluded with a brief discussion of Merton and Nicholson's observations on the appearance of the Balmer series in mixtures of hydrogen and helium, at relatively high pressure.

UNIVERSITY OF CALIFORNIA.

LECTURE ROOM WALL CHARTS.

BY F. J. ROGERS.

LECTURES in physics, particularly in electricity and mganetism, demand more or less profuse illustration by plots and diagrams. This may be secured by blackboard diagrams, lantern slides, or wall charts. The latter have marked advantages, chief of which is the fact that they continue silently to proclaim their messages so long as they hang on the wall.

I have had constructed about fifty charts illustrating all branches of electricity and magnetism. They are uniformly four feet square, and when stored are neither folded nor rolled but are hung compactly on a rack one behind another so that any one can be removed sideways without disturbing any of the others. One or two colors besides black are used when this will contribute to clearness of exposition. To the same end, explanatory words and phrases are printed on the charts so that the meaning of the chart may be conveyed with the minimum of explanation. It has been found that letters an inch square or somewhat less, are amply large enough to be readily visible to a class of a hundred.

The content of the message conveyed by a chart varies more or less continuously between two extremes. In the case of one extreme the chart is expected to present a single fact or principle rather concretely, precisely, and with reference to a real experiment. As an example consider the chart, "Lines of Force of a Magnet." A long slim magnet was laid on a sheet of paper and its field carefully plotted over the whole area, four feet square. The effect of the earth's field was cancelled. The result at first glance is quite familiar, but a second glance shows that many lines near the poles have a double curvature, which I have never seen represented in any published dia-

gram. As a second illustration consider the chart representing "Electric Flux-densities for Equal Voltage Gradients" and "Voltage Gradients Required to Produce Equal Flux Densities." Along with the diagrams representing flux densities for condensers in parallel and voltage gradients for condensers in series are given numerical data for common dielectrics.

Charts representing the other extreme are intended to illustrate a series of related facts, principles, and experiments, as fully as may be done in a single chart. As an example let us consider the chart, "The Effect of Temperature on Resistance." This chart represents graphically the resistance of ten metals, three alloys, carbon, and electrolytes at temperatures from − 180° C. to 500° C. All are assumed to have a resistance of 100 at 0° C. Most pure solid metals are represented by lines very close together, with iron and nickel increasing in resistance considerably faster with temperature than the others. The alloys and carbon are represented by lines which are nearly horizontal while electrolytes are shown to change at a very much greater rate in the opposite direction. The changes of resistance upon melting of Hg., Sn., Pb., Bi., and Zn. are shown. On the charts are the following brief statements: "Melting a metal doubles its resistance, Bi and Sb exceptions." "Glass and many salts become conductors at red heat or below." "All insulators become conductors above 2000° C."

The chart "Vacuum Tube Rays" may be taken as another example of charts with a multiple message. A diagram of tube with a pierced cathode has properly placed a single loop carrying a current, beyond the cathode is a "magnetic screen" and beyond that an electromagnet. The "cathode rays" represented by a series of dots, "Electrons" are curved by the "weak magnetic field" and strike the "Anti-cathode," from which radiate "X-rays" represented as a series of concentric wave crests called "ether waves." Passing through the pierced cathode and slightly deflected by the "strong magnetic field" on the "canal rays" or "positive atoms" which are represented by a series of dots much larger than those representing the cathode rays. On the same chart is a diagram of a "Coolidge tube" showing the "hot cathode" in series with a battery and the tungsten anti-cathode. "Cathode rays" are represented as striking the anti-cathode in a "highest vacuum" and "X-rays" are represented as radiating from the anti-cathode and through the glass walls of the tube.

STANFORD UNIVERSITY.

DAMPED ELECTRICAL OSCILLATIONS.

BY R. B. ABBOTT.

THE subject of "Damped Electric Oscillations" can be treated by the method of analogy. The parallelism between the dynamical and electrical equations for simple harmonic motion and logarithmically damped motion is well known. A parallelism exists between the electrical and dynamical equations for linearly damped motion and for combined linearly and

logarithmically damped motion. The solutions in the last two cases named above are easily obtained, by analogy and a greater simplification is obtained in the solution of the latter case than has hitherto been obtained.

Oscillatory motion is most easily described in terms of the projection of a radius vector r, rotating with constant angular velocity w, about a center c. Let the radius vector make an angle θ with the X-axis, passing through the center c such that $\theta = wt$, where t is the time from the instant when the radius vector coincides with the positive X-axis. The motion may be described as taking place in either X or Y, thus: the displacement $x = r \cos wt$ and $y = r \sin wt$.

The point P at the end of r traces out some curve such as a circle, ellipse or some form of spiral. If there is no dissipation of energy during periodic motion, the auxiliary curves will be of the spiral family: the auxiliary curve for damping proportional to the speed being the logarithmic spiral and for constant damping, the rectilinear (Archimedes') spiral.

The differential equations expressing the acceleration in free periodic motion such as the cases above mentioned, are of the second order with coefficients which are either constants or expressible as functions of the time. If p and q are such coefficients, the general equation is written as follows:

$$d^2y/dt^2 + pdy/dt + qy = 0.$$

By reference to the auxiliary curves, the values of p and q can be determined for all classes of periodic motion, and their differential equations can be written.

In the case of logarithmic damping, r decreases at a rate proportional to itself.

$$\frac{dr}{dt} = -kr,$$

from which $\log r/r_0 = kt$ and $r = r_0e^{-kt}$. Therefore the auxiliary curve is a logarithmic spiral. In the case of rectilinear damping, r decreases at a constant rate,

$$dr/dt = -k_1,$$

from which $r = r_0 - k_1t$.

In the case of combined logarithmic and rectilinear damping, it is easily shown that $r = r_0 - k_1te^{-kt}$.

In each case substitute $y = r \sin \theta$ in the general equation. By comparison with the electrical cases, the constants can be determined in terms of C, L, and R.

WASHINGTON STATE COLLEGE.

A THERMO ELECTRIC MOTOR.

BY PAUL KIRKPATRICK.

THE reaction of thermo-electric currents to a magnetic field is familiarly shown in the radio-micrometer. If the suspended system of the radio-micrometer were free to turn without constraint and the radiant impulses

communicated to its juncture intermittently the coil might be caused to rotate continuously. Such an instrument, which may be called a thermo electric motor, has been constructed.

The armature possesses a number of thermo couples disposed in a circle about the intended axis of rotation. An opaque shade screens half of these while the remainder are exposed to a radiation, say of the sun. Currents traverse a net of wires connecting the couples and, being mounted in the field of a permanent magnet, the system starts to rotate. The successive eclipses of the junctures serve the purpose of commutator and brushes and the forward rotation is maintained.

In the present model there are six silver-German silver junctions and six conductors in the drum armature. The axis of rotation is vertical and the armature carried on a single jewel. The magnet poles are shaped closely about the armature and a fixed iron core protrudes into the latter. The motor will operate under the action of sunshine or any artificial radiation of proper intensity.

UNIVERSITY OF CALIFORNIA.

A WAVE COMPOSITION MODEL.

BY J. G. BROWN.

A MODEL designed to show the simultaneous production of transverse and longitudinal waves was shown at a previous meeting of the Physical Society. The present model, based upon the same principle, shows the composition of two transverse waves in the same plane and also in planes at right angles.

Each component wave is produced by the rotation of a shaft having cranks set at uniform distances and with constant difference of phase along the entire shaft. By acting upon rods, these shafts convey their motions to two rows of balls, the motions being combined in the same direction in the upper row and at right angles in the lower row.

The wave-length of the back component is fixed at 18 inches, but the front component can be varied by changing shafts. Wave-lengths of 18 inches, 12 inches and 9 inches are provided, giving ratios of 1 : 1, 1 : 2 and 2 : 3.

The shafts are connected by chain and sprockets and the relative frequencies are determined by the size of the sprockets. Ratios of 1 : 1, 1 : 2, 2 : 3 and 3 : 4 are provided.

Relative velocities are determined by the combination of wave-lengths and frequencies. The direction of the wave components may be made the same or opposite by means of a reversing gear.

The phase of the wave components can be fixed by the positions of the shafts when the gears are enmeshed.

The model is especially designed for laboratory use, although it will show certain things very well to large groups.

Some of the phenomena which can be illustrated are the following:

1. Variation of amplitude with phase in the interference of similar plane polarized waves.
2. Standing waves. Reflection at a dense medium and at a rare medium.
3. Elliptically and circularly polarized waves.
4. Beats between similar waves.
5. Plane polarized white light in a vacuum.
6. Unpolarized or ordinary white light in a vacuum.
7. Plane polarized white light in glass. Ordinary refraction and dispersion.
8. Ordinary monochromatic light in calcite. Double refraction.

A trace of the motion of any ball can be taken at any time so that the motion of the parts of the medium as well as the distortion of the medium can be studied in detail.

STANFORD UNIVERSITY.

DIURNAL VARIATIONS IN TERRESTRIAL MAGNETISM AND IN THE ELECTRICAL POTENTIAL OF THE EARTH.

BY FERNANDO SANFORD.

IN an article entitled "The Electrostatic Charges of the Earth and Sun and their Relation to Terrestrial Magnetism" which was presented at the Cleveland meeting of the American Physical Society on November 27, 1920, were given some curves showing the diurnal variation of the earth's potential at Palo Alto, California, for the month of August, 1920, and a curve showing the

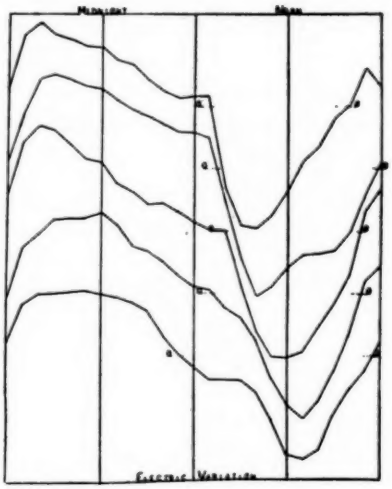

Fig. 1.

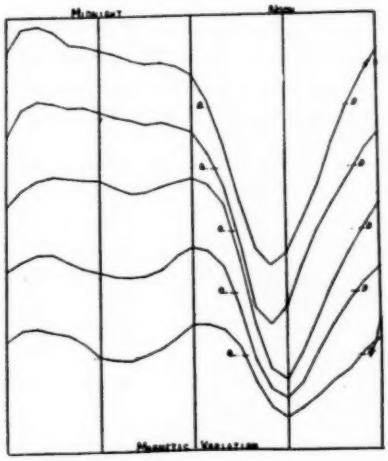

Fig. 2.

mean diurnal variation of the N component of terrestrial magnetism at the Falmouth, England, magnetic observatory for the month of August for the

twelve years, 1891–1902. The measurements of the potential change at Palo Alto have been continued, and in Table I. are given the data expressed in millivolts of the observed change for the months August, September, October, November and December, 1920. Along with these, for purposes of comparison, are given Chree's values of the mean diurnal variation of the *N* magnetic component for the corresponding months for the twelve years above mentioned. The actual numbers given by Chree have all been divided by 5, thus making the magnetic force unit used $5 \cdot 10^{-6}$ C.G.S. unit, instead of $1 \cdot 10^{-6}$ C.G.S.

TABLE I.

Diurnal Variation in Millivolts in the Negative Charge of the Earth at Palo Alto, California, Compared with the Diurnal Variation in the N Component of Terrestrial Magnetism for the Corresponding Months at Falmouth, England, Expressed in $5 \cdot 10^{-6}$ C.G.S. Units.

The positive sign before the quantities in the table of electrical variations does not mean an increase of the positive potential of the earth, but an increase in the negative potential. It is used in this manner because an increase in the negative potential at a given place should cause an increase in the N magnetic component at that place.

Electric Variation.					Hr.	Magnetic Variation.				
Aug.	Sept.	Oct.	Nov.	Dec.		Aug.	Sept.	Oct.	Nov.	Dec.
+ 6.	+ 2.2	+11.8	− 1.2	+ 4.6	6 p.m.	+18.4	+11.4	+ 8.8	8.0	+ 5.6
+23.3	+17.3	+28.3	+14.8	+16.5	7 "	+26.0	+19.8	+13.8	+10.8	+ 9.0
+27.3	+28.7	+33.8	+19.3	+19.9	8 "	+17.2	+22.4	+16.6	+13.0	+ 9.6
+23.3	+30.2	+32.6	+23.8	+20.0	9 "	+25.4	+22.0	+17.8	+13.0	+ 8.2
+21.7	+28.7	+27.7	+24.0	+20.4	10 "	+21.2	+20.4	+17.4	+10.6	+ 7.4
+19.5	+26.7	+23.2	+24.0	+21.0	11 "	+20.6	+20.6	+17.0	+ 8.4	+ 4.6
+19.2	+25.7	+22.0	+25.8	+19.9	12	+19.2	+19.2	+16.6	+ 7.2	+ 0.4
+14.7	+22.2	+15.1	+22.0	+18.9	1 a.m.	+17.8	+17.6	+14.4	+ 5.8	− 0.8
+13.7	+19.2	+12.5	+15.3	+17.2	2 "	+15.8	+16.2	+12.6	+ 5.6	− 1.0
+ 8.7	+16.8	+ 8.5	+11.8	+14.2	3 "	+14.8	+14.8	+12.8	+ 6.0	+ 0.6
+ 5.0	+14.2	+ 9.0	+10.3	+ 6.6	4 "	+14.8	+15.6	+14.6	+ 8.0	+ 2.8
+ 3.0	+12.0	+ 6.2	+ 6.0	+ 0.8	5 "	+13.8	+14.0	+16.0	+12.4	+ 6.0
+ 3.5	+11.7	+ 3.0	+ 2.5	− 3.3	6 "	+11.0	+12.6	+17.0	+15.0	+10.2
+ 3.5	+10.0	+ 1.0	+ 1.5	− 7.3	7 "	+ 2.6	+ 5.2	+15.2	+14.0	+10.2
−26.0	−10.0	+ 0.6	− 4.4	− 7.4	8 "	−12.2	− 7.0	+ 8.6	+ 8.8	+ 8.4
−38.0	−29.3	−17.7	− 7.4	− 7.7	9 "	−30.4	−27.0	− 8.8	− 4.4	+ 2.8
−39.0	−41.0	−32.1	−14.7	−11.9	10 "	−45.4	−44.4	−30.0	−21.0	− 8.0
−34.8	−37.7	−40.3	−27.2	−20.5	11 "	−51.0	−49.8	−44.0	−30.6	−16.2
−27.3	−31.7	−40.8	−35.9	−31.9	12	−46.8	−44.2	−47.2	−33.2	−19.6
−17.5	−27.8	−39.0	−40.2	−33.2	1 p.m.	−36.8	−31.4	−39.2	−28.6	−16.6
−13.0	−27.4	−31.0	−34.4	−30.1	2 "	−26.2	−19.6	−27.4	−19.0	−12.6
− 3.7	−26.3	−23.8	−24.9	−19.3	3 "	−11.6	−10.8	−16.6	−10.4	− 7.8
+ 0.7	−20.2	−14.0	−12.5	−10.6	4 "	+ 1.2	− 3.0	− 7.4	− 3.0	− 4.2
+12.3	− 7.7	+ 6.6	+ 4.0	− 4.7	5 "	+10.8	+ 5.4	+ 1.4	+ 3.8	+ 1.6

In the table, an increase in the negative potential of the earth is marked + and a decrease −, in order to bring them into correspondence with the magnetic data, since an increase in negative potential should cause an increase in the *N* magnetic component.

The two sets of data are also expressed graphically in Fig. 1 and Fig. 2.

PALO ALTO, CALIFORNIA.

Sun Spots and Terrestrial Magnetic Storms.

By Fernando Sanford.

IT has long been known that a close relation exists between sun spots and the irregular magnetic disturbances known as magnetic storms, but no satisfactory reason for this relation has been shown.

The work of Hale and others, at Mt. Wilson, has shown that sun spots are the visible manifestation of tremendous cyclonic disturbances upon the sun. It has also been shown that powerful magnetic fields are produced over large areas in the centers of these cyclonic vortices, which fields are apparently due to convection currents set up by the rotation of negative ions around the storm centers.

Störmer's calculation of the electric and magnetic forces involved in a given sunspot show them to be great enough to produce strong induced currents at the distance of the earth.

It has frequently been shown that magnetic storms have the characteristics which would follow from vortex currents in the earth or in the upper atmosphere. It has been assumed that these currents are due to electrons expelled from the sun and forced into spiral paths by the earth's magnetic field.

The fact that both clockwise and counter-clockwise vortices are shown by magnetic storms in the same regions indicate that their rotational motion is not caused by the earth's magnetic field. Also, it is known that the sun has a magnetic field about 50 times as strong as the earth's field, hence no electrons can be expelled from the sun except in the direction of its magnetic axis, and such electrons would not reach the earth.

The diurnal variations in the earth's electrical potential which have been recorded at Palo Alto seem to show that the earth, as well as the sun, is highly electrified, and with a negative charge. This gives plausibility to the assumption that the other planets are similarly electrified, and furnishes a theoretical basis for the conclusion, hitherto based upon empirical data, that the periodicity of sun spots depends upon the relative positions of the planets. Reference is made to the evidence furnished by La Rue, Stewart and Loewy that the sun spot area varies with the angular distance between Mercury and Venus and to the fact that the mean sun spot period of a little more than 11 years is closely related to an integral number of revolutions of each of the nearer planets.

Thus: Mercury makes 45 revolutions in 11 years;
Venus makes 18 revolutions in 11.08 years;
Earth makes 11 revolutions in 11 years;
Mars makes 6 revolutions in 11.25 years;
Jupiter makes 1 revolution in 11.8 years.

Attention is also called to the fact that the diurnal variation of the N component of terrestrial magnetism has been shown by Chree to be about 70 per cent. greater in the four-year period including sunspot maximum than in the corresponding period at sunspot minimum. This phenomenon can not be

due to the sun spots, themselves, since it is shown by the records of days of least magnetic disturbance. From the point of view of the present writer, such a variation could be caused by the sun only if the total electric charge of the sun varied periodically with the sunspots. Apparently, both the disturbances in the sun's electric field which gives rise to sun spots and the increase in the diurnal variation of the earth's magnetic field must be due to electrical induction by other charges in the solar system.

STANFORD UNIVERSITY.

NOTE ON THE FAHRENHEIT SCALE.

BY FLORIAN CAJORI.

QUOTATIONS were read from Fahrenheit's five papers in the Philosophical Transactions of London, for the year 1724, the three parts describing the thermometers Fahrenheit had used for the ten preceding years. Conclusions: (1) From the first quotation it follows that Fahrenheit used as fixed points the ice-water-salt or sal-ammoniac temperature and blood temperature; the interval was divided into 4×24 or 96 steps; (2) from the second quotation it follows that Fahrenheit (unlike other experimenters) used also a third point, the ice-water temperature, presumably for more perfect checking; (3) from all three quotations it follows that previous to 1724 the boiling point of water was not used by Fahrenheit as a fixed point in the graduation of his thermometers, that continuing his scale upward by equal steps the boiling point of water fell at 212°. On his scale the freezing point of water chanced to come at 32°. It is not known whether or not Fahrenheit changed his fixed points to the freezing and boiling points of water after the year 1724. A disregard of Fahrenheit's own statements has led to many false and contradictory statements, such as: (1) Before 1724 he used 32° and 212° as fixed points in his graduations; (2) the interval between ice-water-salt and ice-water was subdivided by continued bisection into 32° and the scale built up from these degrees; (3) he borrowed the number 96 of his subdivisions from Florentine thermometers; (4) he used as fixed points ice-water-salt and boiling mercury, subdividing the interval into 600 degrees; (5) he regarded his zero as an absolute zero (very probably untrue).

Corroboration of the conclusions of this paper is found in two contemporary writers, namely, C. Wolf in Acta Eruditorum for 1714 and C. Kirch in Miscellanea Berolinensia, T. VI, printed 1740. Both of them owned thermometers made by Fahrenheit.

NATURE OF IONIZATION IN THE POINT TO PLATE DISCHARGE.

BY EVALD ANDERSON.

IT has long been known that in the so-called point to plate, electrical unidirectional discharge the ions are principally of the same sign as the point. In 1911 Zeleny tried some experiments, the results of which indicated that at

moderate currents, and gas pressures above half an atmosphere, very few if any ions of the opposite sign to the point, were present, while at lower pressures, these ions became more and more numerous. Field exploration experiments by H. T. Booth, in 1917 indicated the presence of ions of the opposite kind, but an investigation by Tolman and Carrer gave results which were not in accord with this latter conclusion. However, certain phenomena in connection with so-called electrical precipitation seem to show that at least at the higher voltages ions of the opposite sign are present.

Experiments were made for the purpose of throwing further light on this point. The method consisted in measuring the potential acquired by an insulated wire probe between the point and plate and determining how this potential varied with the potential of the point. The distance between the point and the plate was in all cases 25 mm. The probe was usually a fine wire bent in a small loop. It was carefully insulated from both the point and and plate. The potential was measured by a Braun type voltmeter reading to 5,000 volts, and by an attraction disk voltmeter, reading to 50,000 volts. Since both the potential of the point and the probe were measured on the same instruments the comparative values are correct, and independent of the absolute calibration of the instruments. The gas was air in all the experiments, and the pressure was either 10 or 76 cm. In the former case the air was dried over P_2O_5, but in the latter no drying agent was used.

For the low pressure experiments, a battery of ten 600 volts D.C. generators, connected in series, supplied current at a potential up to 6,000 volts, while for the other experiments a 50,000 volt D.C. generator was used.

At a pressure of 10 cm. the potential of the probe varied directly as that of the point up to the maximum potential tried, 4,800 volts. The arcing potential for this pressure and electrode setting was about 5,600 volts, for either positive or negative point. It is evident from the plotted results that the sign of the point does not markedly influence the character of the curve. The current passing, was, however, approximately three times as large when the point was negative.

The curves for atmospheric pressure are markedly different. In the first place, the arcing potential is approximately twice as high when the point is negative as when it is positive, although the potential at which ionization begins is nearly the same in both cases. Thus the curves for the point negative all show a break at about 30,000 volts, indicating that at this point the relative number of positive ions is suddenly increased. This could be explained by the assumption that at this potential the negative ions had sufficient energy to ionize the air at or in the surface of the plate. This explanation is supported by the phenomena noted when a thin plate of mica was placed on the plate so that its edge was directly under the point and probe. It has long been known that the presence of such a dielectric at the positive terminal causes ionization at this terminal. Such ionization is easily detectable by the characteristic light given off where positive ions are formed. In this case the break

in the curve with the mica on the plate occurred at 24,000 volts, instead of at 30,000 volts, without the mica. The arcing voltage was also correspondingly lowered.

Curves showing the current, and the probe potential as functions of the potential of the point for one position of the probe, with and without mica on the plate indicate that at low potentials, the mica simply acts as an insulator, raising the potential of the probe and lowering the current. At higher potentials when ionization sets in at the edge of the mica, the potential of the probe is lowered, due to the positive ions, and the current is greatly increased. Sharp points, on the plate, as iron filings, have effects similar to an insulator, in that they lower both the arcing potential and the potential of the probe.

With the point positive none of the above effects are noticed. The data taken were not sufficient to definitely prove the shape of the curves in this case, due largely to the small range between the ionization and arcing potentials, but it was definitely proven that neither small points nor insulators at the plate affected the field in such cases. A test was then made on the effect on the arcing potentials of such materials, when the point was positive, and it was proven that not only this was entirely unaffected, but that also the arcing potential for a point and plate with the point positive was exactly the same as for two points the same distance apart. This shows that the phenomena at the positive terminal determined the maximum potential in such a system. It is probable, therefore, that the effect of dielectrics at the positive terminal is due to a modification of the plate into a point or edge, and local disturbance of the field due to the dielectric capacity and charges on its surface.

The behavior at 10 cm. pressure was quite different. Not only was the arcing voltage about the same for either sign, but a thin sheet of mica on the plate had no reproducible effect. It is probable, therefore, that the nature of the ions in a point to plate discharge is markedly different at low pressures, although it is not yet known whether the change is gradual or is abrupt at some one pressure.

These experiments have also shown that the potential acquired by a probe in a point to plate discharge probably is not simply and directly related to the concentration of ions at that point, when the probe is not there. Thus, the arcing voltage between the point and plate in these tests was only 35 kv., with the probe 5 mm. from the plate, while with the probe absent, the arcing voltage was 45 kv. showing that the probe had a decidedly disturbing influence. The probe, in fact, acts as a separate ionization center in the field, which is shown by silent discharge, from the probe to the plate, or sparks to or from the probe at the higher voltages.

UNIVERSITY OF CALIFORNIA.

AN EXPLANATION OF X-RAY DIFFRACTION PATTERNS FROM ROLLED METALS

BY C. T. DOZIER.

WHEN X-rays are passed through a thin sheet of rolled metal a diffraction pattern of two-fold symmetry is formed on a photographic plate set a few centimeters back of the sheet of metal. The diffraction pattern is weaker parallel to the direction of rolling than it is in the direction perpendicular to this. It is shown by photographs that this effect can be duplicated by parallel laying of elongated isometric crystals. The laterally diffracted rays are doubtless less absorbed than those emerging longitudinally. The effect in rolled metals is explained by assuming that the rolling causes the longer dimensions of embedded crystals to be forced into parallel alignment and the X-ray diffraction pattern thus affords a means of determining the direction of rolling in a metal specimen.

UNIVERSITY OF CALIFORNIA.

THE CONCENTRATION OF MONOCHROMATIC X-RAYS BY CRYSTAL REFLECTION.

BY ELMER DERSHEM AND C. T. DOZIER.

IT has previously been shown, Proc. Am. Phys. Soc., PHYS. REV., Vol. XI. p. 244, that a crystal surface if deformed into the shape of a logarithmic spiral of revolution will bring X-rays of a certain wave-length to a focus provided the source of X-rays is at the origin of the spiral.

Such a reflecting surface has now been formed by the use of small crystals of halite. The intensity of the monochromatic X-rays obtained in this way is 10 per cent. of the intensity of the entire radiation of the X-ray tube at the same distance, this intensity being measured with an air-filled ionization chamber.

UNIVERSITY OF CALIFORNIA.

MINUTES OF THE NEW YORK MEETING, FEBRUARY 26, 1921.

THE 108th meeting of the American Physical Society was held in Room 305 Schermerhorn Hall, Columbia University, New York City, on Saturday, February 26, 1921. There was a single session beginning at ten o'clock a.m. with an attendance of about 225. President Theodore Lyman presided.

There was a meeting of the Council held at 2:30 o'clock p.m. on February 26, 1921, in the office of Dean Pegram, of Columbia University. The following elections to membership were made: *Elected to ordinary membership*: Frederick Barry, Richard T. Cox, T. M. Dahm, Marion Eppley, Roy Y. Ferner, Twao Fukushima, Warren K. Green, L. O. Heath, Usaku Kakinuma, Paul M. Mueller, Alexander McLean Nicolson, Simon Sonkin, W. Ewart Williams; *transferred from membership to fellowship*: Alfred H. Bucherer and Frank W. Ham.

At the annual meeting of the Society held in Chicago on December 30, 1920, amendments in the wording of the Constitution and By-Laws were adopted, the sole purpose and effect of which is to change the designations of the two classes of membership formerly named "regular members" and "associate members" to "fellows" and "members," respectively. All persons now designated regular members become fellows without further action, and all associate members become members. There is no change in the conditions for future election to either class of membership nor for transfer. The grade of honorary fellow replaces that of honorary member.

The following program of eighteen papers was presented, three papers being ready by title only:

Reflection of X-Rays from Crystals. W. M. STEMPEL.

The Reflection of X-Rays from Calcite. BERGEN DAVIS and W. M. STEMPEL.

Soft X-Rays of Characteristic Type. E. H. KURTH, introduced by K. T. Compton.

On the Absorption of X-Rays by Chromium, Manganese and Iron. WILLIAM DUANE and HUGO FRICKE.

The Piezo-Electric Resonator. W. G. CADY.

The Binaural Location of Pure Tones. R. V. L. HARTLEY and THORNTON C. FRY.

Note on the Characteristics of the New Singing Tube. CHAS. T. KNIPP. (Read by title.)

Broken Tone From Reed Instruments. JOHN B. TAYLOR.

Electron-Tube Drive for Tuning Forks. E. A. ECKHARDT, J. C. KARCHER and M. KEISER.

A High-Speed Oscillograph Camera. E. A. ECKHARDT.

A Method of Measuring Surface Tension of Liquids. HARRY CLARK.

The Spectral Structure of the Luminescence Excited by the Hydrogen Flame. HORACE L. HOWES.

Photoelectric Phenomena in Coated-Filament Audion Bulbs. ERNEST MERRITT.

Polarization Capacity and Polarization Resistance as Dependent upon Frequency. ERNEST MERRITT.

The Motion of Electrons Between Coaxial Cylinders in a Uniform Magnetic Field. ALBERT W. HULL.

An Electrical Doublet Theory of the Nature of the Molecular Forces of Chemical and Physical Interaction. R. D. KLEEMAN.

The Copernican Atom Radiating Energy.—A Physical Interpretation of Planck's Quantum Rule. ALBERT C. CREHORE. (Read by title.)

A Copernican Atomic Model Based on Electromagnetic Theory. ALBERT C. CREHORE. (Read by title.)

Reflection of X-Rays from Crystals.

By W. M. Stempel.

THE reflection of X-rays from crystals was treated theoretically by C. G. Darwin in 1914 on the assumption that the electrons were clustered so closely about the nucleus of the atom that their distances from the nucleus were small compared to the distance between the planes of atoms in the crystals.

Three years later (1917) A. H. Compton attempted to take into account, theoretically, the effect of assuming that the distances of the electrons from the nucleus was comparable with the distance of the reflecting planes, and arrived at the conclusion, after attempting to derive a general distribution function, that electrons arranged as in the Bohr atom or at least in rings was the only arrangement that would satisfy experimental facts. Both the above authors had to resort to plane waves or their equivalent to get their final workable equations.

The present writer starts with the rings of electrons and plane waves, and in addition carries out the integration in a somewhat different order as follows: A plane is considered in the first or surface atomic layer. This plane is chosen at random and the succeeding planes are taken equal distances apart from this, say D plus E, where D is the distance between the atomic planes of the crystal. The reflection from this series of planes is obtained. The initial plane is then changed and the process repeated. This integration is carried out for all possible positions of the initial plane. The result of this work is as follows:

The intensity of the beam of monochromatic X-rays reflected from a perfect crystal is

$$I_m = I_0 D \left(\frac{e^2}{m_e c^2}\right)^2 \frac{1+\cos^2 2\theta}{2} \frac{1}{E^2} \cdot \frac{1}{1+\frac{\beta^2}{E^2}\delta^2} \cdot e^{-Bn^2} \times \sum_s \left\{ N_s \left(1 + 2m - m(m+1)\frac{D\sin\theta}{2a_s}\right) \left(\frac{\sin \beta a_s}{\beta a_s}\right)^2 \right\},$$

where I_0 is the intensity of the incident beam,

e is the unit charge on the electron (in e^2/mc^2),

e in e^{-Bn^2} in base of Naperian log,

m_e is the mass of the electron,

m is an integer defined by $2a/mD > \sin\theta > 2a/(m+1)D$,

c is the velocity of light,

D is the grating space of the crystal used,

θ is the grazing angle,

E is the fractional part of the energy extracted from the incident beam per atomic layer that it passes through. Here the entire energy is being taken and includes as part both the pure absorption and that part re-radiated by the electron.

β is a constant and equal to $2\pi n/D$,

e^{-Bn^2} is Debye heat factor,

n is the order of the spectrum under consideration,

N is the number of electrons per c.c. belonging to one sized ring,

s refers to the particular electron orbit under consideration, and the summation is understood to be extended to each of the different orbits present no matter to what atom they may belong, the radius of the orbit is all that counts so long as the atom is in the reflecting plane, if not a correction must be made on account of this displacement.

It is to be observed that the intensity of the reflected beam given in equation one is in general not the maximum. Reflection can take place from a system of mathematical planes only when their distance apart is so adjusted as to satisfy the equation, $n\lambda = 2(D + \delta) \sin\theta$. The reflected intensity given by equation (1) is thus the intensity to be expected for any arbitrary wave-length and any arbitrary grazing angle. The maximum value will be reached only when the proper distance between reflecting planes becomes equal to the distance between the atomic planes of the crystal, or $\delta = 0$. The values of the intensity fall off very rapidly on either side of $\delta = 0$.

When a heterogeneous beam is used a small range of wave-lengths will be reflected. That wave-length for which $\delta = 0$ will be most intense, but on either side energy will still be reflected, the intensity falling off rapidly. The range of waves which would have any appreciable value is however very small and (I_0) for such a range is easily considered constant. To get the total energy reflected in the case of a heterogeneous beam we may integrate the equation (1) with respect to δ and get

$$I_{HT} = I_0 D \left(\frac{e^2}{m_e c^2}\right)^2 \frac{1 + \cos^2 2\theta}{2} \frac{\pi}{E\beta} e^{-B n^2} \times \sum_s \left\{ N_s \left(1 + 2m - m[m+1] \frac{D \sin\theta}{2a_s}\right) \left(\frac{\sin \beta a_s}{\beta a_s}\right)^2 \right\}.$$

In all these equations it is to be remembered that the first bracket under the summation sign is never less than one as will be noted on inspecting the condition which determines the value of (m).

If the beam reflected from the one crystal is allowed to fall upon another, placed with its atomic planes parallel with the first, the condition for reflection is correct for every element of the beam and thus serves an ideal method of studying reflection phenomena.

If for two parallel crystals, monochromatic light is allowed to fall upon the first crystal, the intensity of the beam coming from the second is

$$I_{m2} = I_0 D^2 \left(\frac{e^2}{m_e c^2}\right)^4 \left\{\frac{1 + \cos^2 2\theta}{2}\right\}^x \frac{1}{E^4} \frac{1}{1 + \frac{\beta^2}{E^2}\delta_1^2} \frac{1}{1 + \frac{\beta^2}{E^2}(\delta_1 - \delta_2)^2} e^{-2B n^2} \times \sum_s \left\{ N_s^2 \left(1 + 2m - m[m+1] \frac{D \sin\theta}{2a_s}\right)^2 \left(\frac{\sin \beta a_s}{\beta a_s}\right)^4 \right\},$$

where (x), the exponent of the polarization factor, will lie somewhere between

1 and 2, probably not far from 1. Its value will depend upon how completely the beam is polarized by the first reflection. δ_1 refers to the first crystal and δ_2 to the second. We will think of δ_1 being required because the wave-length is not correct for the grazing angle, and δ_2 because the second crystal is not quite parallel with the first.

When heterogeneous X-rays fall upon the first crystal, the total energy reflected from the second for any value of δ_2 will be obtained by integrating this last equation with respect to δ_1. The following equation is obtained,

$$I_{H2} = I_0 D^2 \left(\frac{e^2}{m_e c^2}\right)^4 \left\{\frac{1+\cos^2 2\theta}{2}\right\}^x \cdot \frac{f(\delta_2)}{\beta E^3} \cdot e^{-2Bn^2} \times \sum_s \left\{ N_s^2 \left(1 + 2m - m[m+1]\frac{D\sin\theta}{2a_s}\right)^2 \left(\frac{\sin\beta a_s}{\beta a_s}\right)^4 \right\},$$

and the values of the $f(\delta_2)$ are given below for various values of δ_2 where $A = E/\beta$.

δ_2	0	.2A	.4A	.6A	.8A	1.0A	1.4A	1.8A
$f(\delta_2)$	1.571	1.555	1.510	1.441	1.354	1.257	1.054	.868
δ_2	2.0A	2.4A	2.8A	3.2A	3.6A	4.0A	5.0A	8.0A
$f(\delta)_2$	.785	.644	.531	.443	.371	.306	.218	.090

To get the fractional part of the beam that is reflected at the second crystal we have only to divide the last equation by the second and we get

$$D\left(\frac{e^2}{m_e c^2}\right)^2 \left\{\frac{1+\cos^2 2\theta}{2}\right\}^{x-1} \frac{f(\delta_2)}{\pi E^2} e^{-Bn^2} \times \sum_s \left\{ N_s \left(1 + 2m - m[m+1]\frac{D\sin\theta}{2a_s}\right)\left(\frac{\sin\beta a_s}{\beta a_s}\right)^2 \right\}$$

The fractional part of the beam reflected may be easily obtained for all the other cases.

The theory has the advantage of disclosing the following facts which are new to X-ray reflection theory:

The reflected energy does not approach infinity for either large electron concentration or small grazing angles.

The theory definitely discloses the variation of the intensity of the reflected beam for conditions deviating from those for perfect reflection, which variation measures the sharpness of interference patterns. This sharpness of interference patterns is independent of electron distribution, a fact shown some years ago by Debye.

The theory discloses many other interesting facts too complex to point out here. One may be stated because of its extreme importance, and that is that for polished or irregular crystals the intensities of successive orders vary inversely as the square of the order, a fact shown experimentally to be true by the Braggs (they no doubt worked with polished crystals).

The possibilities of the theory together with some interesting experimental conformation will appear later when the details of this work are published.

POLARIZATION CAPACITY AND POLARIZATION RESISTANCE AS DEPENDENT UPON FREQUENCY.

BY ERNEST MERRITT.

THE success of the experiments described by Gen. Squier, in which radio frequency signals were sent along bare wires immersed in water, makes it reasonably certain that the electrolytic capacity of such wires is much less at high frequencies than for a slowly changing charging potential, and preliminary measurements at the Bureau of Standards have confirmed this conclusion. I have recently made determinations of the electrolytic capacity of platinum in sulphuric acid through a wide range of frequencies and find that in this case also the capacity depends upon the frequency. Platinum electrodes in sulphuric acid were chosen for test in order to make possible a quantitative comparison of the results with those predicted by theory. The electrodes were of wire 0.41 mm. in diameter and 18 mm. long. As measured by ordinary charge and discharge the charging potential being 0.1 volt or less, the capacity of the cell was approximately 5 microfarads.

In determining the capacity at different frequencies an alternating current of the frequency desired was developed by an oscillating audion, and a loosely coupled secondary circuit was tuned to resonance by its adjustable condenser. When the electrolytic cell was placed in series with the variable condenser it was necessary to make a small change in the setting in order to restore resonance; from this change and the original capacity the electrolytic capacity could be computed. When the cell was not used its place in the circuit was taken by a "dummy" having the same inductance and resistance. For measurements in the audible range a standard mica condenser was used. Frequencies were determined by a wave meter in the case of radio frequencies and in the case of lower frequencies by comparison with tuning forks. Current measurements were made with a crystal detector in an untuned circuit loosely coupled to the secondary.

The results obtained are in the main in agreement with the simpler form of the Warburg diffusion theory. For example, the capacity of the cell (two electrodes in series) was found to be 2.37 mf. for $n = 1{,}810$ and 0.12 mf. at $n = 1{,}000{,}000$. If the capacity were proportional to $1/\sqrt{n}$ throughout this whole range as predicted by the Warburg theory, the value for $n = 1{,}000{,}000$ should be 0.10 mf. In the range $n = 4{,}000{,}000$ to $n = 500{,}000$ the capacity appears however to be proportional to $1/n$ instead of $1/\sqrt{n}$. There is some reason to expect such a change in the law at high frequencies, since the theory is based upon the assumption that the ions are so numerous that equilibrium between the ionic concentration in the liquid and that in the electrode can be established instantly and without appreciable change in the concentration in

the liquid. At sufficiently high frequencies this assumption must cease to be justified.

CORNELL UNIVERSITY.

PHOTOELECTRIC PHENOMENA IN COATED FILAMENT AUDION BULBS.

BY ERNEST MERRITT.

IN audion bulbs having oxide-coated filaments a film often forms on the plate, sometimes appearing as an irregular discoloration of the polished surface, but often scarcely visible. It has been pointed out by Case that these films show photoelectric activity. In repeating the experiment described by Case I find that the activity, although smaller than that of a sodium or potassium cell of the Elster and Geitel type, is sufficient to permit of convenient measurement by means of a moderately sensitive galvanometer. The active rays appear to be at the violet end of the spectrum and in the near ultraviolet. The current produced by light from an open arc is reduced by about 20 per cent. by a piece of glass 5 mm. thick. Owing to the high vacuum the voltage current curve is remarkably flat; complete saturation is reached at about 10 volts and increase in voltage to 120 volts produces no further change in current.

Since the active films are produced by the evaporation of the oxide coating on the filament it seemed reasonable to expect that the filament would also show photoelectric activity I find this to be true in the case of all the tubes tested. With the plate and grid connected and maintained at a positive potential the current obtained by illuminating the filament is much smaller than when the plate is made negative with reference to grid and filament. In one case, for example, it was found that $i = 5 \times 10^{-9}$ for filament negative and $i = 4 \times 10^{-8}$ for plate negative. But the difference is not as great as the difference in area would lead one to expect.

The current obtained when the filament is illuminated increases greatly when the temperature of the filament is raised. To study this temperature change the effect of illumination was measured for different values of the filament current, the thermionic current being first measured and then the increase due to illumination. The photoelectric current, *i.e.*, the increase in current due to illumination, was found to increase steadily with the temperature, although less rapidly than the thermionic current. At a dull red heat the photoelectric current was in one case found to be 1,400 times as great as that from the same filament when cold. The photoelectric current measured was in this case 7 microamperes. With the same illumination and applied E.M.F. an Elster and Geitel potassium hydride cell, set at the angle of incidence showing maximum effect, gave a current of 0.34 microamperes. This result is all the more remarkable when it is remembered that the area illuminated in the case of the potassium cell must have been at least a hundred times greater than the area of the filament. Increased photoelectric activity at high temperatures has also been observed by Case with barium cells. While the

results may prove to be due to secondary causes there is every indication that we have to deal with a real increase in photoelectric activity.

When polarized light was used there was no indication of any difference in the magnitude of the effect for different positions of the plane of polarization, either with the filament or plate.

Coated filament tubes, probably because of the active film referred to above, have been found to show readily the phenomena that result from secondary emission from the plate. With the grid maintained at a positive potential of 120 volts with respect to the filament the curve of plate potential-plate current is the typical dynatron characteristic described by Hull. In one of the tubes used in these experiments (a Western Electric "V" tube) the plate current reaches a positive maximum at about 70 volts, falls again to zero at 94 v., reverses and reaches a negative maximum at 97 v., passes through zero a second time at 107 v. and then continues on the positive side of the axis. The emission of secondary electrons from the plate is especially strong in the range between the positive and the negative maximum. In order to determine whether the secondary emission was influenced by light the plate voltage was set at the point corresponding to the first zero, *i.e.*, on the "negative resistance" part of the characteristic, and the plate was then illuminated. The effects observed were at first erratic, the deflection observed being sometimes positive and sometimes negative, depending upon the tube used, and occasionally changing from positive to negative with the same tube. This erratic behavior was found to be due to the fact that the effect of illuminating the plate is opposite to that of illuminating the filament, the former being negative, the latter positive. The two effects could be separated by using a narrow beam of light.

The effect of illuminating the plate is in the direction corresponding to increased electron emission from the plate. In some cases however the effect appears to be greater than would result from the simple addition of the photoelectric effect to the secondary emission.

When the filament alone is illuminated the deflection is positive, indicating a diminished secondary emission from the plate. The effect is probably due to the increased space charge near the filament, which reduces the velocity of the electrons reaching the plate. Experiments intended to test this explanation are not yet completed.

CORNELL UNIVERSITY.

THE REFLECTION OF X-RAYS FROM CALCITE.

BY BERGEN DAVIS AND W. M. STEMPEL.

THE reflection of X-rays of various degrees of homogeneity from calcite was measured by means of a double X-ray spectrometer. The experiments covered a wave-length range from .3 to .8 Å. The radiation from a Coolidge tube under accurate control after passing through the slits of the spectrometer fell on the first crystal designated by *A*. A portion of the general

radiation ($\Delta\lambda$) reflected from crystal A fell on a second crystal designated by B. Crystal B was placed so that its reflecting planes were parallel to those of A. A very great reflectivity was observed at this position of parallelism. The reflectivity was measured by comparing the ionization produced by the bean ($\Delta\lambda$) of X-rays before and after reflection from Crystal B.

The reflection obtained from B depended on the degree of homogeneity of the beam reflected from A. The reflectivity of B thus depended on the character of crystal A. The crystals were consequently investigated in combinations or pairs. The reflectivity depended also on the similarity of the crystals and on the perfection of the crystals and on the state of polish of their surfaces.

The pairs of crystals investigated are designated as follows:

($A_1 - B_1$): A_1 was a specimen of Montant calcite, not quite clear. Defects were visible in the interior. B_1 was a clear specimen of Iceland spar. The surfaces of both crystals were polished. ($A_2 - B_2$): A clear specimen of Iceland spar was split along a cleavage plane. The two surfaces were polished. The crystals were so mounted that the reflecting surfaces were the surfaces that had been contiguous before splitting. ($A_3 - B_3$): A clear specimen of Iceland spar was split along a cleavage plane. The surfaces were not polished. The crystals were so mounted that the reflection took place from the planes that were contiguous before cleavage. Maximum reflection was obtained when the several pairs of crystals were mounted with the reflecting planes parallel. Some reflection was obtained when crystal B was turned slightly out of parallelism with A. Interesting curves of energy distribution were obtained by rocking crystal B through small angles each side the position of maximum reflection. These curves differed in width. The dissimilar pair ($A_1 - B_1$) gave much wider rocking curves than the others. The curves were quite narrow for the more perfect pair ($A_3 - B_3$). The width at half maximum in this case was about 16″ of arc. A table is given of the per cent. of the energy reflected from crystal B for each pair. The results indicate that the reflection of homogeneous radiation from a perfect crystal of calcite (were such possible) would be greater than 50 per cent.

Angle.		Percent Reflection.		
		$A_1 - B_1$.	$A_2 - B_2$.	$A_3 - B_3$.
3	.317	25.8	37.3	
3–30	.369	24.6		
4	.422	23.4	33.6	44.4
5	.527	21.5	30.5	
5–30	.580	20.8		43.1
6	.632	19.8	27.9	
6–30	.686			42.8
7	.737	16.7	26.	
7-30	.790			42.8
8	.842	17.3	24.6	

PHOENIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY.

SOFT X-RAYS OF CHARACTERISTIC TYPE.

BY E. H. KURTH.

THE following investigation was undertaken in the hope that, if special care were exercised to secure a high vacuum, troublesome ion effects experienced by earlier investigators, making the use of a window imperative or the interpretation of results uncertain, would disappear.

The apparatus used was of the type generally employed, the radiation being studied through its photo-electric action upon a metal plate. The target and filament were sealed into the small end of a large conical glass tube and a platinum detecting disk was supported at the other end. In the body of the cone proper a series of plates were arranged diverging from the target to the disk, and they were connected alternately together.

In order to secure as high a vacuum as practicable, the apparatus was joined by means of large bore tubing to a condensation pump. An appendix containing a small quantity of charcoal was also attached to the tube. Electric heaters were employed to bake out the apparatus for several hours at about 400 degrees before making a run. Finally at the end of this period the vapor traps and charcoal were immersed in liquid air.

The detecting disk was connected to an electrometer with a sensitivity of about 1,700 millimeters per volt.

The intensity of the measured effect was found to be essentially independent of potentials up to 800 volts which were applied to the diverging plates to prevent the passage of ions from one end of the tube to the other. Variations in the strength of a magnetic field applied perpendicularly to these plates likewise had no effect. However a potential of 135 volts on the plates and a weak magnetic field were applied during all the runs.

Three elements have thus far been tested, aluminium, iron and carbon. The radiation was found to be entirely characteristic of the particular element used, and was in every sense reproducible.

With aluminium very definite breaks were found in the intensity curve at 38 and 120 volts, indicating three types of radiation corresponding to 326 Å.U. and 103 Å.U., respectively. These values may be uncertain within possibly 5 per cent. It is likely that there is additional radiation shorter than 133, but, if so, it is not strong enough to be detected when superimposed on the effects of the longer wave-lengths.

It seems possible that 103 may be the L series, and 326 may be the M series.

The radiation curves of iron show very definite breaks at 62.8 Å.U., 48.4 Å.U. and 16.3 Å.U. The two long wave-lengths seem to represent the extremes of the M series for iron while the 16.3 break probably corresponds with the L series lines of the element. The radiation effect set in at approximately 25 volts and there was some indication of a break in the curve at 50 volts corresponding to 247 Å.U.

With the element carbon a break at 43.6 Å.U. was very pronounced. This agrees exactly with the extrapolated value for the K series within the limits of

error of the extrapolation. No certain indication of any further breaks which might belong to a longer wave-length series have yet been observed. The radiation set in initially at approximately 20 volts.

This work is at present being extended to cover some of the other elements including beryllium.

PALMER PHYSICAL LABORATORY,
PRINCETON, NEW JERSEY.

ON THE ABSORPTION OF X-RAYS BY CHROMIUM, MANGANESE AND IRON.

BY WILLIAM DUANE AND HUGO FRICKE.

IN the experiments reported in this paper we have measured the K critical absorption wave-lengths for chromium, manganese and iron. The object of the research has been not only to obtain accurate values for these particular absorption wave-lengths, but also to collect evidence bearing on the following general questions:

(*a*) Has the nature of the chemical compound containing a chemical element any influence on the critical absorption wave-lengths characteristic of that element?

(*b*) Does a critical absorption wave-length represent a single sharp boundary separating wave-lengths of X-rays that are more absorbed from those that are less absorbed, or does it consist of several such boundary lines lying close together?

Evidence has been obtained by means of photographic spectrometers indicating that black phosphorus has a K critical absorption differing slightly from that of yellow phosphorus.[1] Further, on general theoretical grounds, we would expect energy changes due to transfers of electrons in atoms to depend to some extent upon the forces binding the atoms together; and, since the frequencies of vibration of X-rays are proportional (theoretically) to the energy changes, the X-ray wave-lengths should vary, therefore, with the nature of the chemical bonds.

In measuring critical absorption of light elements by means of photographic spectrometers, photographs have been obtained[2] that show certain irregularities between the light and the dark areas. These irregularities were interpreted as indicating a certain "structure" in the absorption limit. Attention was called to the fact that they could be explained by supposing that the critical absorption corresponds to the transfer of an electron from an inner orbit to any one of a number of possible orbits outside of the periphery of the atom, each one of these outside orbits representing an absorption line.

In our measurements we have used an ionization spectrometer with a calcite reflecting crystal. The general arrangement of the apparatus and the mode of

[1] J. Bergensen, Comptes Rendus, Oct. 4, 1920, p. 624.

[2] Hugo Fricke, PHYS. REV., Sept., 1920, p. 202; and G. Hertz, Zeits. für Physik, Vol. 3, 1920, p. 19.

procedure have been explained in several publications from this laboratory.[1] We have made only one change in the instruments. The X-rays now emerge from the X-ray tube through a very thin mica window fastened at the end of a long side tube, which projects out from the X-ray tube very near to the crystal. This reduces the absorption of the X-rays by the walls of the tube and by the air, and greatly increases the intensity of the X-rays in the long wave-length region of the spectrum. The rays leave the target making a very small angle with its surface, and the width of the slit is such as to define a beam of rays that has angular breadths ranging from 5′ to 9′ of arc in different experiments. These breadths are of the same order of magnitude as those used in the photographic work.

The absorbing screens are made by dipping a very thin sheet of paper into a solution containing the chemical element to be investigated, and then allowing it to dry. The deposit of salt obtained in this way is not very uniform. The screens are placed in the path of the X-rays before they strike the crystal. The potential on the tube is so chosen that we do not have the spectrum of second order.

The curves in the lantern slide shown represent the ionization current as a function of the angle that fixes the position of the crystal. A peak appears on one of these curves, which represents the longest L emission line (Ll) in the X-ray spectrum of the tungsten target. This line is quite faint and appears to be complex.

The sharp drops in the curves indicate the positions of the critical absorption. It is possible to estimate the values of the critical absorption wave-lengths to within considerably less than 1/20 of one per cent.

The curves represent the critical absorption of iron, manganese and chromium. The values of the wave-lengths are:

	Fe	Mn	Cr
$\lambda \times 10^8$ cm. =	1.7377	1.8893	2.0623.

We used two salts of iron, Fe_2O_3 and $FeSO_4$ $(7H_2O)$, and could detect no difference between their critical absorption wave-lengths. This result indicates that the energy changes corresponding to the critical absorption must be sensibly the same for the iron atoms in the two states of chemical combination respectively.

All of the curves have sharp drops corresponding to the K critical absorption wave-lengths, but none of them show more than one such drop, nor do they indicate any irregularity in the neighborhood. In other words, there appears to be no evidence for a "structure of the absorption limit."

The failure of the ionization method to indicate an absorption structure can not be attributed to a lack of resolving power, for, as stated above, the breadth of the absorption drop is about the same as in the photographic measurements.

Irregularities in the thickness of the absorbing screen would tend to obliterate evidence as to the structure of the limit. Our screens, however, were about as

[1] PHYS. REV., Dec., 1917, p. 624; ibid., Dec., 1919, p. 516; ibid., Dec., 1920, p. 526.

uniform as those used in some of the photographic experiments. They were usually just thick enough to cut down the intensity on the short wave-length side of the drop to about one half that on the long wave-length side.

The Piezo-Electric Resonator.

By W. G. Cady.

A PLATE or rod suitably prepared from a piezo-electric crystal, and provided with metallic coatings, can be brought into a state of vigorous resonant longitudinal vibration when the coatings are connected to a source of alternating E.M.F. of the right frequency. Under these conditions, the plate reacts upon the electric circuit in a remarkable manner.

The vibrations are a consequence of the so-called "converse" piezo-electric effect, *i.e.*, the deformation resulting from an electric stress; while the periodically strained condition of the plate, through the action of the "direct" effect, sets up a periodic component of polarization which causes the reaction referred to.

On the theoretical side, the phase and magnitude of the counter-polarization in the plate are derived, as well as expressions for the current flowing to the plate and for the total current in the oscillatory circuit. The plate is assumed to be in parallel with the circuit capacity. It is shown that, owing to the piezo-electric polarization, and to the absorption of energy in the plate, the apparent electrostatic capacity and resistance of the plate are not constant, but depend upon the frequency somewhat as does the motional impedance of a telephone receiver. Over a certain range in frequency the capacity may even become negative.

A graphical method is developed for presenting the results of the theory. By an application of this method, it is possible, after making a series of purely electrical observations, to deduce the coefficient of viscosity of the material, even though the absolute value of the piezo-electric constant is not known.

Methods are described for mounting a small plate of piezo-electric crystal upon a rod of any solid elastic substance in such a manner as to excite longitudinal vibrations in the entire rod. The high-frequency viscosity of the rod can then be found, subject, however, to more or less error due to losses in the cement which attaches the crystal plate to the rod.

Experiments on longitudinal vibrations in rods or plates of steel, quartz, and Rochelle salt are described.

The possibility is discussed of using the piezo-electric resonator for a standard of high frequency; for excluding from a circuit oscillations of a given frequency; and as a coupling device to transfer small amounts of power from one circuit to another at a particular frequency.

Wesleyan University, Middletown, Conn.,
February 3, 1921.

THE BINAURAL LOCATION OF PURE TONES.

BY R. V. L. HARTLEY AND THORNTON G. FRY.

THE theoretical explanation of experiments on the apparent location of sounds has been very largely confined to the relatively simple case of pure tones where the phenomena are almost entirely binaural. The theories advanced have been quite successful in explaining the results of that type of experiment in which the intensities at the two ears are kept equal and the direction of the image is observed for various phase differences. For those experiments in which the phases are kept equal and the intensity ratio is varied no satisfactory explanation has so far been offered.

The present paper presents a theory which when applied to the latter type of experiment explains why the observer has difficulty in assigning a definite direction to the image and why different observers get widely different results. The results given by Stewart[1] for this type of experiment are shown to be in entire agreement with the present theory. When the theory is applied to experiments with equal intensities and varying phase difference it indicates why the results of different observers are in substantial agreement with one another and why the theories advanced have explained them successfully even though certain factors have been neglected.

Experiments are suggested in which the phase difference and intensity ratio are adjusted simultaneously in such a manner as more nearly to approach the conditions of every day experience. To facilitate such experiments computed curves connecting the polar coördinates of a source with the phase difference and intensity ratio of the resulting stimulus are given for sounds of representative frequencies. Emphasis is placed upon the importance of recording the apparent distance as well as the direction of the sound image.

RESEARCH LABORATORIES OF THE AMERICAN TELEPHONE & TELEGRAPH CO.
AND THE WESTERN ELECTRIC COMPANY, INC.,
February 10, 1921.

NOTE ON THE CHARACTERISTICS OF THE NEW SINGING TUBE.

BY CHARLES T. KNIPP.

THE temperature difference that is necessary to cause the new singing tube to emit a tone, when the portion *B* (Fig. 1) is kept at room temperature while the tip *A* is heated, was observed to be about 400° Centigrade. If,

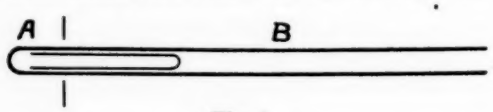

Fig. 1.

New singing tube.

however, *B* is cooled to the temperature of liquid air the temperature difference

[1] PHYSICAL REVIEW, May, 1920, p. 425.

necessary is greatly reduced, being about 200° Centigrade.[1] The pitch is also considerably lowered.

It was deemed desirable to make quantitative measurements of these temperature differences and also of the corresponding pitch of the tone emitted. To this end B was held successively at different temperatures, ranging from that of liquid air to values considerably above room temperature, while A was heated electrically in each case to a temperature where a tone was emitted continuously. The two portions A and B were each housed within a separate copper tube of about 1 cm. wall thickness and heavily insulated with asbestos. The temperatures were accurately measured by means of thermo-junctions,—three being attached to A and three to B. The temperature control of each section was good and could be held fairly constant at will.

TABLE I.

No.	A Average Temp. in Degrees C.	B Average Temp. in Degrees C.	Total Temp. Difference in Degrees C.	Temp. of B in Absolute Measure.	Vibrations per Second.
1.....	1	−181	182	92	213
2.....	204	− 88	292	185	300
3.....	355	− 16	371	257	378
4.....	448	+ 26	422	299	425
5.....	524	+ 57	464	330	450

The results from the only run thus far made are contained in Table I. In observation No. 1 the part B was placed within a glass jacket heavily wrapped

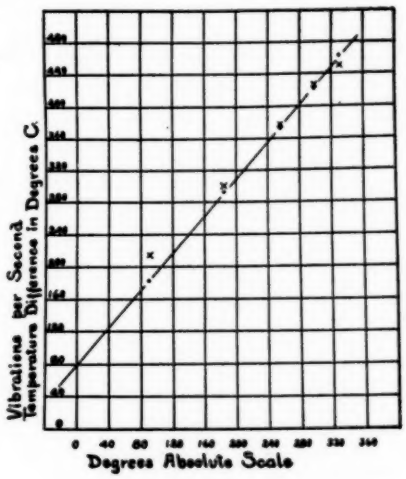

Fig. 2.

Characteristics of the new singing tube.

with aspestos and cooled directly to − 181° C. by means of liquid air. The temperature of A was allowed to fall until the tone emitted was just main-

[1] PHYS. REV., N.S., Vol. XV., p. 336.

tained. This by repeated trials was found to be at 1° C. Thus the temperature difference when B was cooled to $-$ 181° C., for this particular tube, was found to be 182° C. In observations 2 and 3 the part B was placed in a special copper tube designed by the author[1] some years back for the determination of intermediate temperatures. Observation 4 was for B at room temperature (note that it was now necessary to heat A to 448° C.), while in No. 5 the part B was warmed up to 57° C., and the tip heated electrically to 527° C. before the tube responded.

The absolute temperatures of the part B are given in the second last column, while the corresponding vibrations per second are listed in the last column. These data are represented graphically in Fig. 2, in which the total temperature differences as ordinates are plotted against absolute temperatures. The relation is strictly linear except possibly for the last reading at 330° absolute temperature. By extending the straight line to the left we are able to determine the temperature difference that should maintain the tone when B is cooled to absolute zero. For this particular tube the graph shows this temperature difference to be about 80° C.

The same figure also shows the corresponding vibration frequencies (indicated by crosses) plotted to the same scale against absolute temperatures. This relation also seems to be linear except for the point taken at 91° absolute. The pitch was determined by means of a tone variator.

Lastly the vibration frequencies and temperature differences in degrees centigrade, as shown by the graphs, are nearly equal numerically. This, however, should be considered as a coincidence.

Observations with tubes of different pitches and extending over wider temperature ranges are under way.

UNIVERSITY OF ILLINOIS,
February 10, 1921.

BROKEN TONE FROM REED INSTRUMENT.

BY JOHN B. TAYLOR.

IN musical instruments of the wood wind group, the pitch of the tone is generally considered as determined by the distance from the mouthpiece to the nearest of the side openings or vents which are uncovered. The opening or closing of vents below the first has some influence in modifying the pitch and quality.

If the vent hole nearest the mouthpiece is small, and located in a proper proportional relation to the total length of the air column, a harmonic tone is determined, in which case there may be considerable variation in the size and location of the harmonic vent without appreciably affecting the tone.

If the harmonic vent occupies a position intermediate to those positions proper for two harmonics, other factors may determine which one of the two

[1] PHYS. REV., Vol. XV., p. 125.

harmonics will be heard—but, generally, whichever one starts to sound will continue so long as there is no change in the manner of blowing.

The bassoon differs from the other wood wind instruments in a large degree because the theoretical locations of the ventages are much more widely separated than the spread of the finger tips. This difficulty was met in the early days before the development of key mechanisms by leaving an unusually thick wall on the wooden tube and boring the long ventages of small size at an angle. As a result, the upper and lower portions of the instrument are more "closely coupled" than is the case with the other wood wind instruments such as flute, oboe and clarinet, and the uncovering of a single vent does not approximate an open-end condition of the air column.

While experimenting with unusual cross fingerings, several combinations have been found producing tones quite foreign to the normal characteristic voice of the instrument. The term "broken tone" has been selected as best describing these unusual sounds.

One of these broken tones has been selected for illustration and wave form records have been made. The particular tone is that produced by sounding the low F (approximately 87 cycles). If while this F is sounding, the third finger of the left hand is raised, uncovering the vent which is ordinarily considered as that through which the tone d (approximately 145 cycles) speaks, the complex tone results.

A skilled observer judged the complex tone to be composed of B^b (116 cycles), f (174 cycles) and a differential resultant tone of BB^b (58 cycles).

Such tones would result if the third finger left hand vent (called d) is regarded as being near enough to the octave position for the low F (87 cycles), tending to produce the octave f (174 cycles.) Also, if the d vent is sufficiently long and restricted to produce B^b at 116 cycles, when the air column is regarded as ending at the outside of the third finger d vent, the second member of the complex tone may be explained.

The broken or somewhat rattling character of the tone was judged to indicate that the mode of vibration alternated rapidly between that producing B^b as a fundamental, and that giving f as a second harmonic. The rapid alternation of the two produces the low pitched differential tone BB^b.

The photographic records of the sound wave are considered to be in agreement with this suggested explanation. The complex or broken tone is not easy to decipher because the normal single tones of the bassoon are rich in harmonics and of complicated shape.

Schenectady, N. Y.,
February 11, 1921.

Electron Tube Drive for Tuning Forks.

By E. A. Eckhardt, J. C. Karcher and M. Keiser.

Eccles and Jordan and Abraham and Bloch have recently described methods for maintaining the vibrations of tuning forks by the use of electron tube circuits. Regenerative circuits are used, the coupling between

the grid and plate circuits being provided by the tuning fork. There is no appreciable electrical or magnetic coupling.

It was found by the authors that this method of driving a tuning fork does not yield large amplitudes unless the circuit is carefully designed. If the circuit is properly adjusted for one electron tube it will frequently work less well with another of the same type and sometimes not at all.

A more flexible driving arrangement was devised by the authors which permits the driving of a given fork with any tube of a given type, provision having been made for the slight readjustments which are necessary when the tubes are changed. The adjustments are such that even the transition from one type of tube to another can be made in a few minutes.

In our driving arrangement the tuning fork does not provide the coupling between the two branches of the regenerative circuits. When the circuits are closed the fork driving magnet is energized and the fork is displaced from its equilibrium position. This sets up a small initial vibration of the fork which induces in the plate circuit a small E.M.F. of the fork frequency and hence an alternating component of the plate current. The plate circuit is coupled loosely to the grid circuit. If the mutual inductance between the grid and plate circuits is of the proper sign the alternating current of the fork frequency will be regeneratively amplified and the periodic driving forces on the fork will increase giving rise to larger fork amplitudes. The limit is set primarily by the operating characteristics of the electron tube.

A transformer may be inserted in the plate circuit and the arrangement may then be used as a generator of feeble alternating currents. By proper design of the circuits a very satisfactory wave-form can be obtained. The tuning fork acts as an automatic frequency control of high precision.

We believe that the apparatus is likely to be quite useful for the precise measurement of electrical quantities which are a function of the frequency.

BUREAU OF STANDARDS,
WASHINGTON, D. C.
February 11, 1921.

A HIGH-SPEED OSCILLOGRAPH CAMERA.

BY E. A. ECKHARDT.

A HIGH-SPEED oscillograph camera was developed in our Sound Laboratory which has the advantage of being available for use in connection with any type of oscillograph. It has been used with G. E. oscillographs, an Einthoven galvanometer, and to record various purely mechanical vibrations.

An aluminum drum 5 feet in circumference and 4 inches wide is arranged to have a film of similar dimensions stretched over its convex surface. This drum is mounted in a light tight housing, which is something over 8 inches wide, to which access is obtained by means of a sliding door. The shaft protrudes from

the housing on both sides. One end of it carries the driving pulley which is belted to a motor, the other carries a phosphor bronze worm which engages an idling cast iron wheel. In contact with the flat face of this wheel are the pole faces of an electromagnet which is energized only when an exposure is being made. During the exposure this electromagnet grips the cast iron wheel and prevents its rotation. Consequently the worm feeds along the wheel and the drum executes a spiral motion, the pitch of which is determined by that of the worm and the wheel. With a spiral of ½ inch per revolution nearly eight full revolutions are obtained on the film. This gives a film record 40 feet long. Several worm and wheel combinations can be mounted interchangeably on the drum, which permits the obtaining of longer and narrower records.

The shutter consists of a thin steel blade which covers the shutter opening. A plunger attached to the blade is pulled into a controlling solenoid against a spring when exposure is to be made. When the shutter solenoid is deënergized the shutter is closed by the action of this spring.

The shutter and spiral initiating solenoids are connected in series. They are energized simultaneously therefore and the shutter opens and the drum begins to spiral at the same moment. This operation can be controlled by a relay and the performance may therefore be subjected to remote control.

When a record is to be made the drum is brought to the desired speed. By closing the control key the shutter is opened and the drum begins its spiral motion. A contact is whipped open by the driving pulley when the drum has spiralled the desired distance. The shutter closes and the gripping magnet releases the cast-iron wheel which again begins to idle. A spring then restores the drum to its initial position in the housing.

Records have been obtained in which the 40 feet of film covered a time period of less than one second. With suitable time index marks and a steady rotational speed of the drum, time intervals may be read to one millionth of a second.

Bureau of Standards,
Washington, D.C.,
February 11, 1921.

A Method of Measuring Surface Tension of Liquids.

By Harry Clark.

MANY good methods of measuring surface tension require either such elaborate mathematical treatment or such expensive apparatus that they cannot be used in elementary laboratory courses, and none of the other methods are quite satisfactory. The method here described requires little apparatus, is simple mathematically, and gives fairly accurate results.

The apparatus consists of a glass tube, perhaps 2 cm. in diameter and 10 cm. long, one end of which is provided with a metal plug held in place by a bit of rubber tubing. A fine hole is drilled axially through the plug, which is

turned down so that it terminates externally in a small tube about 3 mm. long and of diameter appropriate to the liquid used—about 0.75 mm. for water.

If liquid is poured into the glass tube, held vertically in a clamp, a drop will form on the end of the metal tube, the horizontal diameter of the drop being equal to the external diameter of the tube. As more water is added, the drop increases in size vertically until it becomes hemispherical (to an excellent degree of approximation) beyond which it becomes unstable and falls off. The surface tension is given by

$$S = rdg\left[\frac{h}{2} + \frac{r}{3}\right],$$

where h is the maximum height of the liquid column above the end of the metal tube; d, the density of the liquid; r, the radius of the metal tube; and g, the acceleration of gravity.

The liquid is introduced slowly and continuously through a funnel, the tube of which is drawn down very fine and lies against the inner surface of the large glass tube. Heights are measured on a paper scale behind the tube, or better still, on graduations etched on opposite sides of the tube. The diameter of the metal tube is measured with a micrometer gauge. No cleaning is required beyond wiping the external edge of the metal tube with a cloth. The error in the results is proportional to that of the measurements, which need not exceed 2 per cent.

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH.

THE SPECTRAL STRUCTURE OF THE LUMINESCENCE EXCITED BY THE HYDROGEN FLAME.

BY HORACE L. HOWES.

THE discovery of the luminescence of many substances when partially bathed in the hydrogen flame but at temperatures below red heat has recently been made by Prof. E. L. Nichols. The present paper represents only one of several tests which have recently been developed to prove that this new type of luminescence exhibits properties similar to that excited at room temperatures by the iron spark, the cathode rays, etc. To this end calcium oxide under hydrogen flame excitation was found to yield the same series of bands as calcite under iron spark excitation and three samples of the Lenard and Klatt phosphorescent sulphides under the two distinct modes of excitation yielded similar series but the distribution of intensity was very different. A new Hilger spectro-photometer was employed with the specimen mounted on a rotating disk of brass gauze and the analysis of the spectra was rather difficult because of very extensive overlapping of the bands.

NEW HAMPSHIRE College,
February 9, 1921.

The Motion of Electrons between Coaxial Cylinders in a Uniform Magnetic Field.

By Albert W. Hull.

THE thermionic current between a straight filament and a concentric cylindrical anode is unaffected by a uniform magnetic field parallel to the filament so long as this field is smaller than a critical value, but falls abruptly to zero when the field is increased beyond this value. The same is true if the outer cylinder is cathode and the inner anode, except that the fall is less abrupt.

The relation between these quantities is very simple in practical cases.

The equations of motion of the electrons, using cylindrical coördinates r, θ, and z, are:

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = F\frac{e}{m} - H\frac{e}{m}r\frac{d\theta}{dt}, \tag{1}$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = H\frac{e}{m}\frac{dr}{dt}, \tag{2}$$

$$\frac{d^2z}{dt^2} = Z\frac{e}{m}, \tag{3}$$

where F and Z are the electric intensities in the radial and axial directions respectively, and H the magnetic field, which is uniform and parallel to the filament. F is a function of r only.

The first integrals of these equations are readily obtained, giving the velocity components dr/dt and $r(d\theta/dt)$. The critical magnetic field, at which the electrons are just able to reach the cylinder, is found either by placing the radial velocity component equal to zero; or by equating the square of the tangential velocity component, multiplied by half the mass, to the total work done on the electron by the radial electric force, *i.e.*,

$$\frac{dr}{dt} = 0,$$

or

$$\frac{1}{2}m\left(r\frac{d\theta}{dt}\right)^2 = eV. \tag{4}$$

This gives

$$V = H^2\frac{e}{8m}\left(R^2 - 2r_0^2 + \frac{r_0^4}{R^2}\right) + H\frac{r_0v_0}{2}\left(1 - \frac{r_0^2}{R^2}\right) - \frac{m}{2e}\left(u_0^2 + v_0^2\left[1 - \frac{r_0^2}{R^2}\right]\right),$$

where R and r_0 are the radii of outer and inner cylinders respectively, V the potential difference between them, u_0 and v_0 are the radial and tangential components, respectively, of the initial velocities of the electrons, and H the magnetic field that is just sufficient to prevent electrons reaching the anode.

If the electrons start from a filament of radius r_0 small compared with R, and the initial energies $\frac{1}{2}(m/e)u_0^2$ and $\frac{1}{2}(m/e)v_0^2$ are small compared with V,

this becomes:

$$H = \sqrt{8\frac{m}{e}}\frac{V^{1/2}}{R}. \tag{5}$$

This is the case of a straight filament cathode in the axis of a cylindrical anode.

If the outer cylinder is cathode and the inner anode, eq. (4) becomes:

$$H = \frac{r_0}{R^2}\sqrt{\frac{8m}{e}V} + 2\frac{mv_0}{eR}. \tag{6}$$

This applies to a helical filament with anode wire in its axis. It applies also to the motion of positive ions produced by electrons from a straight filament in the axis of a cylindrical anode. In this case R is the distance from the axis at which the ions are produced, and e/m the ratio of charge to mass of the ion.

If the radii of both cylinders are large compared with their distance d apart Eq. (4) becomes:

$$V = H^2\frac{e}{2m}d^2 + Hv_0d - \frac{u_c^2}{2\frac{e}{m}}. \tag{7}$$

This is the case of plane parallel plates with electric field normal and magnetic field parallel to the planes.

The approximate paths of the electrons can be calculated for the case of the straight filament and external concentric anode, on the assumption that the space charge distribution is the same with magnetic field as without. This assumption is justified by the experimental fact that the positive ionization, as measured by a central negatively charged electrode (the filament is a small helix in this case, with collecting electrode in its axes) is unchanged by a magnetic field, even when the current to the anode is cut down to less than 1 per cent. of its maximum value by the field. The equation of the path on this assumption is:

$$r = R\left[\sin\frac{2}{3}\theta\right]^{3/2}. \tag{8}$$

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.,
February 10, 1920.

AN ELECTRICAL DOUBLET THEORY OF THE NATURE OF THE MOLECULAR FORCES OF CHEMICAL AND PHYSICAL INTERACTION.

BY R. D. KLEEMAN.

THE forces of interaction between molecules which give rise to the internal heat of evaporation of liquids, their surface tension, etc., are explained by means of the electric forces associated with each atom on account of the electrons and positive electrical charges it contains. Whatever the arrangement of electrical charges their effect at a point some distance from the atom could be represented by an electrical doublet. Now we may arrange a number

of doublets in space so that they attract each other in all directions, by placing them at the points of intersection of three sets of parallel equidistant planes which are at right angles to each other, so that opposite charges face each other in a row of doublets, and the order of the charges is different in two adjacent rows. When the doublets are in motion it would follow from the principle that *the potential energy of a system tends to a minimum* that the doublets would undergo motion of translation and rotation in such a manner that on the average attraction between the doublets would result, and that it would be as large as possible. Thus though any direction of the axis of a doublet in space might be equally probable, attraction would result through an association of the directions of the various doublets. The attraction between two molecules in a liquid would accordingly vary inversely as the fourth power of their distance of separation, and vary directly as the product of the moments of the molecular doublets, multiplied by a factor which would depend on the nature of the interaction of the molecules, and hence depend on the temperature of the liquid, the distance of separation of the molecules, etc.

It can be shown that it follows from the stopping power of the α particle that the moment of the doublet of an atom is proportional to the square root of its atomic weight, and that the moment of a molecule is equal to the sum of the moments of its atoms. This is confirmed by the results of previous investigations by the writer on the law of attraction giving rise to the internal heat of evaporation of liquids, etc., according to which the attraction an atom exerts is proportional to the square root of its atomic weight.

UNION COLLEGE,
February, 1921.

A COPERNICAN ATOMIC MODEL BASED ON ELECTROMAGNETIC THEORY.

BY ALBERT C. CREHORE.

THE atomic model applied to the hydrogen atom is represented in meridian section in Fig. 1. The nucleus, C, is relatively enlarged, it being impractical to represent nucleus and electron to the same scale. The effective nucleus consists of two charges, namely the nucleus, C, with positive charge $2e$ and one negative electron, A or B, of charge e which are inseparable, thus making an effective nuclear charge of plus e. Thus from the neutral atom only one electron is detachable. Both electrons, A and B, in the normal state of the atom at the zero of temperature are in contact with the positive charge, C, one at each pole. All three charges rotate about a common axis, PQ, but the electron has no tendency to describe an orbit revolving around the nucleus on the Saha theory. The rotating nucleus, so far as great distances are concerned as I have shown elsewhere,[1] may be replaced by an equivalent closed circular charged rotating ring.

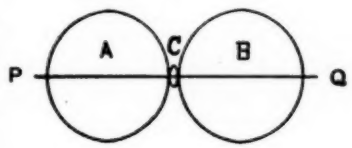

Fig. 1.

[1] Abstract, PHYS. REV., Feb., 1921, p. 252.

The precise state of motion of rotation of the nucleus has not been rigorously solved on the basis of Saha's fundamental equations, but a state of rotation of the nuclear charge, which is supposed uniformly distributed throughout the volume as in the "solid" Lorentz electron, is not incompatible with the theory. It is not necessary to suppose that the angular velocity of an element of charge is constant for all radii; it may increase with the radius. It follows from the theory that two coaxial rings revolving in the same direction exert an accelerating effect upon each other when the distance between centers is several diameters of the rings. This problem is managable because the series employed remains convergent. The convergence disappears, however, as the rings are brought together to within a fraction of the radius, and an exact solution without resorting to these series must apparently be found before the mutual effect of two elementary rings within the body of the nucleus becomes known.

Assuming that a state of rotation meets the theoretical requirements for equilibrium, the spherical charge at rest is unstable, and becomes a pure fiction. This consideration makes possible an approximation to the shape of the positive charge in advance of an exact solution. The potential energy of a uniformly distributed spherical charge, E, at rest is[1]

$$W = 3E^2/5ak. \tag{1}$$

The mass[2] of the charge is

$$m = \frac{4}{5ak}\left(\frac{E}{c}\right)^2. \tag{2}$$

Eliminating the quantity $E^2/5ak$ between (1) and (2) gives the potential energy of the charged sphere at rest as

$$W = \frac{3}{4}mc^2. \tag{3}$$

Einstein and others have adopted mc^2 rather than $mc^2/2$ as the total energy associated with a mass m. In view of (3) the figure $mc^2/2$ cannot represent the total energy, and allowing for an equilibrium rotation the $3mc^2/4$ should be increased by the energy due to the rotation. Why is not the $mc^2/4$ the energy due to the rotation? For, together with (3) the total energy becomes mc^2. Let it be assumed that this is the case, and also that the shape of the sphere is distorted into an oblate spheroid when in equilibrium rotation. The pressure per unit area of surface of the sphere at rest is given by Schott[3]

$$P = 3E^2/8\pi a^4 k.$$

Surface pressure and a tension along the surface are mathematically equivalent and convertible by the formula

$$T = P/\frac{1}{r}\left(+\frac{1}{r'}\right) = \frac{a}{2}P,$$

T being the tension and r and r' the two radii of curvature of the surface, which

[1] Schott, Electromagnetic Radiation, Eq. (381).

[2] Loc. cit., Eqs. (379), (380).

[3] Loc. cit., Art. 257.

merge into a when the surface is spherical. Let it now be supposed that the sphere is distorted by the rotation into an oblate spheroid, both the volume of it and the tension remaining constant. The work required is that to stretch the surface over a larger area, say A. The work is, therefore, AT, and this is to be equated to $mc^2/4$, giving

$$AT = A\frac{a}{2}P = 3AE^2/16\pi a^3 k = mc^2/4. \qquad (4)$$

Whence, by (2) and (4)

$$A = \frac{16}{15}\pi a^2.$$

The area of a sphere, being four great circles, must be stretched according to this result 16/15 of a great circle. If the shape is an oblate spheroid, this fact determines the eccentricity and the major and minor axes. The result is the eccentricity

$$e = 0.945,$$

and the ratio of the axes

$$a/b = 3.058.$$

Numerically, setting $E = 2e = 2 \times 4.762\ 10^{-10}$ E.S.U., and $m = 1.659 \times 10^{-24}$ grams, (2) gives the radius of the sphere of the nucleus at rest as

$$r = 4.86 \times 10^{-16} \text{ cm.}$$

The equatorial radius of the spheroid is, therefore, 7.22×10^{-16} cm., and the polar radius 2.20×10^{-16} cm.

In a former communication[1] the attraction between two such atoms as in Fig. 1 was given when the rotating nucleus is replaced by an equivalent ring of radius a_H, the frequency of rotation being $2K$, twice the Rydberg constant. When the atoms are given all orientations the force comes out an attraction equal to the known gravitational attraction, provided the ring has a radius

$$a = 6.4 \times 10^{-16} \text{ cm.}$$

This ring falls within the body of the rotating nucleus, being less than the equatorial radius 7.22×10^{-16} above. It is not possible to say how to calculate a ring equivalent to the spheroid above until the angular velocities of the elements at different radii are known or obtained from the Saha theory.

The ring equivalent to the negative electron, also a rotating body, is much larger in diameter, and has a linear speed about 1/30.4 part of the ring of the nucleus, and on this account has a small almost negligible gravitational effect.

The natural free period of the nucleus, when suddenly deformed in shape, as by a collision with one of it's own electrons, probably corresponds to the greatest frequency of any known body, as it is also the body of smallest known dimensions. Its natural frequency may be given by setting the whole energy content equal to $h\nu$, giving

$$h\nu = mc^2 \text{ or } \nu = 2.27 \times 10^{23} \text{ frequency.}$$

[1] Loc. cit.

This corresponds to a wave-length of

$$\lambda = c/\nu = 13.2 \times 10^{-13} \text{ cm.}$$

This appears to be the order of magnitude of the wave-length to be expected. The vibration implies a periodical interchange between the rotational energy and the potential energy due to the changing shape of the nucleus.

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THE COPERNICAN ATOM RADIATING ENERGY. A PHYSICAL INTERPRETATION OF PLANCK'S QUANTUM RULE.

BY ALBERT C. CREHORE.

LET Fig. 1 of the preceding abstract represent an atom of hydrogen gas subjected to bombardment by a stream of electrons. To study the effect of a passing electron, begin by neglecting the rotation of the parts of the atom. The mechanical force upon the atom due to the moving electron may be divided into two parts, (1) the electrostatic part, and (2) the part arising from the velocity. The electrostatic force upon one electron in the atom due to the passing electron, barring an actual collision which is not necessary to account for the radiation, can never be as great as the electrostatic force exerted by the rest of the atom upon its own electron, because of the comparatively great distance to the passing electron. Hence, the electrostatic part (1) of the mechanical force alone can never pull an electron away from the atom.

The part (2) due to the velocity of the passing electron is very much less than the electrostatic force, unless the electron moves with a velocity very close to that of light. The experimental velocity observed to produce radiation is of the order 2×10^8 cm./sec. (11 volts) in hydrogen, a speed less than one hundredth that of light. Hence, no such low speed electron will ever succeed in separating an electron from the atom according to theory so long as the rotations of the parts of the atom are neglected. The chief effect of these forces (1) and (2) is to increase the pressure of one electron against the nucleus and diminish that of the other, thus rendering one more readily detachable than the other.

The key to the behavior of this atom is the rotation of the nucleus coupled with its elongated[1] shape. The passing electron exerts a couple on the nucleus tending to turn the direction of its axis of rotation. This effect is due to the term $(\mathbf{q}_1 \cdot \mathbf{q}_2)\mathbf{R}$ in the Saha ponderomotive force equation. Referring to Fig. 2, let PQR be the path of the electron passing the atom H. At the nearest point Q, where the effect is a maximum, this term gives a force shown by the arrow, A, directed toward Q, the vector $\mathbf{R}$ joining the two points. This being the force upon an element in the upper side, the force upon a corresponding element

[1] See abstract, "A Copernican Atomic Model Based on Electromagnetic Theory."

of the rotating nucleus diametrically opposite is reversed in direction and represented by the arrow, B, directed away from Q, the two forces forming a couple tending to turn the axis of rotation.

When the effect of this couple is small, the nucleus may turn a little and roll smoothly without losing contact with the electrons at any time. Then, as the passing electron leaves, this motion rights itself thus restoring all of the energy received back to the passing electron again, which departs with as much energy as it had on arrival. It is considered that the energy of the

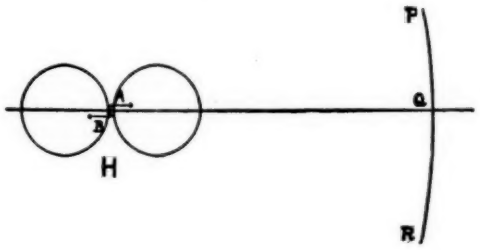

Fig. 2.

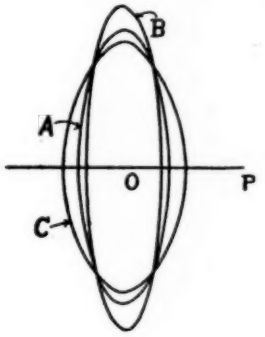

Fig. 3.

passing electron must be sufficient to cause the nucleus to roll over at a rate sufficient to make it leave contact with and bump against or collide with one of its own electrons before energy may be said to be received by the atom.

This process may be likened to the rolling of an elliptical wheel along a horizontal table. When the velocity of the wheel exceeds a certain critical value, the wheel leaves contact with the table and bumps along each half revolution. This analogy also points to the existence of a critical velocity.

These collisions set up the high frequency vibrations characteristic of the nucleus,[1] and the energy thus absorbed is not returnable to the passing electron as it was before, being transformed in character. It is postulated in advance of a rigid proof that this high frequency energy is absorbed by the nearest charges and is effectually imprisoned within the atom, very little, if any, of it being radiated into space as high frequency waves. The only way open to the atom under these circumstances to get rid of this energy is to eject an electron. How this takes place is made clear by electromagnetic theory.

Picture the nucleus thus set into high frequency vibration something like Fig. 3. The axis of rotation, OP, is supposed to remain fixed while the meridian section normally represented by the curve, A, becomes elongated into B and foreshortened into C as extreme distortions during a complete cycle. The motion is probably not as simple as this, but any complexity in the vibration does not affect the argument, for the best that can be expected is to obtain an approximation to the effect that such a vibration may produce. The area of the surface undergoes a periodical variation, and there must occur a corresponding periodical exchange of rotational energy with the potential energy

[1] Loc. cit.

due to the shape. But, the whole energy content of the nucleus takes part in this exchange, and the available energy is, therefore, large.

To approximate the force upon an electron in the immediate vicinity due to this vibration, divide the nucleus into two halves by the equatorial plane, and replace one half by a single equivalent point charge on the axis, say at the center of volume. This point will then oscillate along the axis OP during a complete cycle, since the center of volume moves along this line. The whole nucleus may thus be replaced by two point charges vibrating along the axis, first from and then toward each other, let us say harmonically. The average mechanical force that two such vibrating point charges exert upon an electron moving in its vicinity, together with the force exerted by the second electron inseparably associated with the nucleus, may represent the whole average force that the remainder of the atom exerts upon an electron ejected from itself. This force derived from the Saha theory is

$$F = \frac{e^2}{kr^2(1-\beta_1^2)^{1/2}}[1 - 2\beta_2^2(1 - \tfrac{1}{2}\sin^2\alpha)]. \qquad (1)$$

Let E_1, Fig. 4, represent the position of the ejected electron experiencing this force, and E_2 the position of the nucleus from which E_1 has become detached.

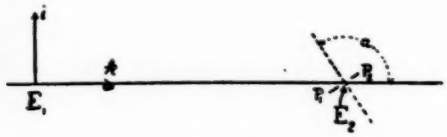

Fig. 4.

The broken line, P_1P_2, represents the direction of the axis, OP of Fig. 3, and the two point charges equivalent to the nucleus are supposed to move back and forth along these lines, P_1 and P_2. When the axis is directed toward the electron E_1, $\sin\alpha = 1$, and (1) reduces to

$$F = \frac{e^2}{kr^2}\frac{1-\beta_2^2}{(1-\beta_1^2)^{1/2}}, \quad \text{polar force.} \qquad (2)$$

When E_1 is in the equatorial plane of E_2, $\sin\alpha = 0$, and (1) becomes

$$F = \frac{e^2}{kr^2}\frac{1-2\beta_2^2}{(1-\beta_1^2)^{1/2}}, \quad \text{equatorial force.} \qquad (3)$$

In these equations $\beta_1 c$ is the velocity of the electron E_1, and $\beta_2 c$ the maximum velocity of the two harmonically vibrating points substituted for the nucleus. When the expression is positive the electron E_1 is attracted toward the nucleus. The whole force acts along a radial line, there being no component perpendicular to the line joining the nucleus to the electron.

Equation (2) states that the force is always an attraction when the nucleus points its pole toward the electron E_1, for it may be assumed that β_2 cannot exceed unity. Equation (3) states that the force may be a repulsion provided β_2 exceeds $\frac{1}{2}\sqrt{2}$. Moreover, this repulsion may be very great indeed if the speed of the electron, $\beta_1 c$, approaches the velocity of light, because of the factor

$(1 - \beta_1^2)^{1/2}$ in the denominator. Although the mass increases with the speed, so also does the force in a similar fashion, and it should not be surprising if velocities approaching that of light are obtained. In fact, the force due to such a vibrating nucleus more than counterbalances the immense electrostatic force normally holding the electron in contact with the nucleus, and will in some cases succeed in ejecting an electron from the atom.

It also appears from (1) that there exists a radius at an angle, α, along which the force is zero, provided β_2 exceeds $\frac{1}{2}\sqrt{2}$. Equating (1) to zero gives this angle

$$\sin^2 \alpha = 2 - 1/\beta_2^2. \tag{4}$$

If the nucleus could turn its axis to this critical direction, the force upon the electron would be zero, and it would consequently move away with a uniform velocity. There would then occur no loss of energy by radiation, because no energy is radiated by uniform motion in a straight line. Under these circumstances the electron would carry away from the atom the whole of its own kinetic energy, and by this means the atom would lose far more energy than it normally does.

Regarding the nucleus and its electron, namely the effective nucleus, as a single physical system, like all physical systems left to themselves the entropy tends toward a maximum. In this instance the system strives to get rid of energy at a maximum rate, and would succeed in increasing the rate if the axis of the nucleus could turn immediately to the critical angle expressed by (4). The mutual forces between it and the electron govern the motion, and it must be expected that the pole of the nucleus will oscillate about this critical angle of zero force, being damped because of the loss of energy by radiation into space from the moving electron. The acceleration of the electron at the distance, x, should, therefore, be expressed by an equation of the type

$$\frac{d^2x}{dt^2} = k_1 \nu e^{-\nu t} \cos \nu t, \tag{5}$$

which multiplied by the mass is the force upon the electron. For, this force oscillates harmonically about a zero value, changing from repulsion to attraction, and decreasing in amplitude as the energy of the high frequency vibration is spent in the secondary process of radiation from the moving electron. The frequency $\nu/2\pi$ expressed by (5) refers to the oscillation of the pole of the nucleus. The whole motion is determined by this equation. A first integral with respect to time gives the velocity.

$$\frac{dx}{dt} = \tfrac{1}{2} k_1 e^{-\nu t}(\sin \nu t - \cos \nu t) + k_2. \tag{6}$$

Now the total energy radiated by an electron moving with this velocity may be calculated from the Liénard expression for the rate of radiation, which for straight line motion is

$$\dot{R} = \frac{2e^2}{3kc^3} \frac{\left(\frac{d^2x}{dt^2}\right)^2}{(1-\beta^2)^3}. \tag{7}$$

The Saha expression for the rate of radiation is not yet known to the writer. Assuming that the electron goes out and back along a radius, the constant of integration becomes zero, because the velocity is zero when the time is infinite. The whole energy lost by radiation in one excursion out and back is then

$$E = \int_0^\infty \dot{R}dt = \frac{\pi e^2 k_1^2}{2kc^3}\left(1 + \frac{23}{90}\frac{k_1^2}{c^2} + \cdots\right)\nu', \tag{8}$$

where ν' is the frequency corresponding to the angular velocity ν in (5), and k_1 is constant by hypothesis. Consequently the whole coefficient of the frequency is a constant, and (8) exactly expresses Planck's quantum rule, radiated energy is equal to a constant times the frequency.

The value of Planck's constant, h, cannot be determined from this, but an estimate of the initial velocity of the electron may be obtained required to make (8) equal to $h\nu'$. The series in the parenthesis may have any value from 1 to infinity according to the value of k_1, and by (6) $k_1/2$ represents the initial velocity. Denoting the sum of this series by S, its value must be $S = 138.2$ to give the proper value of h, this value being

$$h = 2\pi e^2 S/ck. \tag{9}$$

This sum shows that the initial velocity must be very close indeed to that of light.

To obtain an estimate of the relative energies, the radiated energy and the kinetic energy of the electron at some point, consider the maximum velocity possessed by the electron, for example, on its return trip to the nucleus. This velocity is 3.23×10^9 cm./sec., not far from one tenth the velocity of light. The kinetic energy is approximately $47. \times 10^{-10}$ ergs. The maximum energy ever radiated from a hydrogen atom is $h\nu = hK = 0.2154 \times 10^{-10}$ ergs, and the kinetic energy just computed is about 220 times this maximum radiated energy.

The energy lost by the whole atomic system, however, is no greater than the transformed energy received from the passing electron, since the kinetic energy of its own electron is all returned again to the atom. Thus it becomes manifest that the internal energy of the nucleus must have been drawn upon temporarily to provide this large amount of kinetic energy for one of its own electrons. The available energy in the nucleus is $mc^2 = 1.49 \times 10^{-3}$ ergs. This conception of the process involved in radiation illustrates the trigger action, which becomes possible because the energy received from the passing electron is entirely transformed in character, the atom itself impressing upon all subsequent events its characteristic properties.

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